Measurement of the Foreign Exchange and Sales Tax Externality on the Value Added of Labour

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by

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The purpose of this note is to measure the externality created by the existence of sales taxes and trade distortions when labour is taken away from the rest of the economy for use in the project under examination. This externality is calculated as a proportion of the value added of labour estimated by the gross-of-income-tax wage component of the social opportunity cost of labour.

This estimation is made by employing the same general equilibrium model and data base used to calculate the shadow price of foreign exchange for Canada.[1] The solution of the model in this case is to measure the economy-wide change in the value of sales tax and trade distortions per dollar of labour-value added removed from the rest of the economy for a project. To calculate the difference between the shadow price of foreign exchange and its market price the model was solved for the change in the total value of sales tax and trade distortions when an additional unit of imports is required for a project or an additional unit of export is generated by a project.

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1. Glenn P. Jenkins, "Theory and Estimation of the Social Cost of Foreign Exchange Using a General Equilibrium Model With Distortion in All Markets", a paper prepared for Department of Regional Economic Expansion, December, 1976.

I The Model

The model is applicable to a country which is open to trade, has no influence over the world price of the goods it imports or exports, and operates with a flexible exchange rate.[1] There are three production sectors: importable goods, exportable goods and non-traded goods sector. Production is characterized by competition with full employment of factors.

Because of the small country assumption, the world prices of the import and export goods are fixed but their domestic prices will vary with the exchange rate. If w_1 is the world price of the importable good expressed in units of the foreign exchange currency and (E_m) is the (floating) market exchange rate, the domestic price of the traded good in the absence of tariffs or export subsidies is expressed as $D_1 = E_m w_1$. The presence of tariffs on imported goods drives a wedge between their value to the consumer in domestic currency and the domestic currency value of the foreign exchange used to purchase the goods. Let τ_1 denote the ad valorem tariff rate of the imported good. In the absence of other taxes or subsidies the domestic price of the imported good will be $D_1 = E_m w_1 (1+\tau_1)$. If this tariff is not prohibitive, then domestic production of these goods will expand to the point where the marginal resource cost of production of the importable good will also be equal to $E_m w_1 (1+\tau_1)$. For the purposes of this analysis it is useful to separate the domestic consumer's evaluation of the importable good (D_1) into its foreign exchange cost $(E_m w_1)$ per unit, if imported, and the tariff $E_m w_1 \tau_1$ per unit.

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1. The assumptions of flexible exchange rates and fixed world prices are not required in this approach to the calculation of the shadow price of foreign exchange, but they are the most realistic assumptions for the case of the Canadian economy.

Sales taxes levied on the domestic consumption of both domestically produced and imported goods have also an effect on the demand for foreign exchange and thus the market exchange rate. Imposing a rate of sales tax of (t_1) results in a decrease in the quantity demand of both imports and import substitutes and increases the difference between the consumer's evaluation of the imported goods he purchases and the foreign exchange cost of $E_m w_1$ of the goods. As excise taxes are levied on the imported good, its domestic price as seen by the final consumer is $D_1 = E_m w_1 (1+\tau_1)(1+t_1)$ and the price received by the domestic producer of the importable good is $P_1^s = E_m w_1 (1+\tau_1)$.

The domestic price of exportable goods can also be derived from their world prices (w_2). For these goods there is also a rate of sales tax of t_2 on their domestic consumption and a rate of subsidy of k_2 on their total domestic production. The domestic price that consumers will pay for the exportable good will be equal to $D_2 = E_m w_2 (1+t_2)$ and producers will receive $P_2^s = E_m w_2 (1+k_2)$ per unit for the output. The value of $E_m w_2 (1+k_2)$ also represents the marginal resource cost of an additional unit of production.

In this model we wish to express the quantities of all goods in terms of a number of units of foreign exchange. Therefore, w_1 and w_2 will take values equal to 1, giving us consumer and producer prices for the importable and exportable good as follows:

<u>Importable Good</u>

Consumers demand price $D_1 = E_m(1+\tau_1)(1+t_1)$ (1)

Producers supply price $P_1^s = E_m(1+\tau_1)$ (2)

<u>Exportable Good</u>

Consumers demand price $D_2 = E_m(1+t_2)$ (3)

Producers supply price $P_2^s = E_m(1+k_2)$ (4)

In the non-traded good sector we assume that no subsidy exists in its production. However, a rate of sales tax of t_3 is levied on the consumption of these goods so that the consumers pay a price of $D_3 = P_3^s(1+t_3)$ and producers receive (P_3^s) per unit.

Production technology in each of the three sectors is characterized by a homogeneous of degree one production function using two factors of production. In each sector one of the factors used in production is assumed to be fixed in quantity for the time period that we are using this estimate of the shadow price of foreign exchange. These fixed factors are denoted as F_1, F_2 and F_3 in the importable, exportable and non-traded goods sectors, respectively. The other input (V) is used for production in all three sectors. It is assumed to be mobile between the three sectors but fixed in total supply in the economy. The production possibilities can be summarized as follows:[1]

$$\left.\begin{aligned} X_1 &= I(F_1, V_1) \\ X_2 &= E(F_2, V_2) \\ X_3 &= N(F_3, V_3) \end{aligned}\right\} \quad X_j = X_j(F_j, V_j) \qquad \begin{aligned} &(5) \\ &(6) \\ &(7) \end{aligned}$$

where F_1, F_2 and F_3 are fixed factors in their respective sectors and V is the only mobile factor.

If a_{ij} represents the quantity of factor i, (i = 1, 2, 3, V) required per unit of output from sector j, (j = 1, 2, 3) the full employment conditions can be set out as in eqs. (8) to (11).

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1. This model is similar in structure to an earlier two good three factor model developed by Ronald Jones, "A Three-Factor Model in Theory, Trade and History", in Bhagwatti et al. (editors), <u>Trade, Balance of Payments and Growth</u>, North-Holland, 1971.

$$\left.\begin{aligned} a_{11}\ X_1 &= \bar{F}_1 \\ a_{22}\ X_2 &= \bar{F}_2 \\ a_{33}\ X_3 &= \bar{F}_3 \end{aligned}\right\}\quad a_{jj}\ X_j = \bar{F}_j \qquad (8)\text{–}(10)$$

$$a_{V1}X_1 + a_{V2}X_2 + a_{V3}X_3 = \bar{V} \tag{11}$$

where $\bar{F}_1$, $\bar{F}_2$, $\bar{F}_3$ and $\bar{V}$ are the initial endowments of each of the factors in the economy. With competition economic profits will be zero. Thus, if R_i is the "rental" return for the use of one unit of factor i and P_j^s is the price received by producers of good j we will have the following relationships:

$$\left.\begin{aligned} a_{11}\ R_1 + a_{V1}\ R_V &= P_1^s \\ a_{22}\ R_2 + a_{V2}\ R_V &= P_2^s \\ a_{33}\ R_3 + a_{V3}\ R_V &= P_3^s \end{aligned}\right\}\quad a_{jj}R_j + a_{Vj}R_V = P_j^s \qquad (12)\text{–}(14)$$

In this model, the input coefficients a_{jj} are also determined in competitive equilibrium so as to minimize unit costs by the relative "rental" returns of the factors used in each sector. This relationship can be expressed as follows:

$$a_{ij} = f\left(\frac{R_i}{R_V}\right) \tag{15}$$

or

$$\frac{a_{Vj}}{a_{ij}} = g\left(\frac{R_j}{R_V}\right) \qquad \text{where } g' > 0 \tag{16}$$

From equations (8) to (15) we see that factor prices and output quantities cannot be derived from the knowledge of output prices (P_1^s, P_2^s, P_3^s) alone. In addition the endowments of the three fixed factors ($\bar{F}_1$, $\bar{F}_2$, $\bar{F}_3$) and the variable factor ($\bar{V}$) must be known. By solving equations (8) to (10) for each X_j and substituting into equation (11) we obtain equation (17).

$$\frac{a_{V1}}{a_{11}} \bar{F}_1 + \frac{a_{V2}}{a_{22}} \bar{F}_2 + \frac{a_{V3}}{a_{33}} \bar{F}_3 = \bar{V} \tag{17}$$

From equation (16) the changes in the ratio of the input coefficient are expressed as functions of relative factor prices as follows:

$$d\left(\frac{a_{Vj}}{a_{jj}}\right) = \frac{\partial (a_{Vj}/a_{jj})}{\partial (R_j/R_V)} d(R_j/R_V)$$

$$\therefore \quad \sigma_j = \frac{\partial (a_{Vj}/a_{jj})}{\partial (R_i \;/\; R_V)} \frac{(R_j \;/\; R_V)}{(a_{Vj}/a_{jj})} > 0$$

$$\therefore \quad d\left(\frac{a_{Vj}}{a_{jj}}\right) = \sigma_j \frac{a_{Vj}}{a_{jj}} (\hat{R}_j - \hat{R}_V) \tag{18}$$

$$\sigma_j = \frac{\hat{a}_{Vj} - \hat{a}_{jj}}{\hat{R}_j - \hat{R}_V}$$

where σ_j denotes the elasticity of substitution (defined positively) between inputs i and V in sector j and a "^" over a variable denotes the relative change in that variable, e.g., $\hat{R}_V = (dR_V/\ R_V)$. By differentiating equation (17) we obtain:

$$d\bar{V} = \sum_{j=1}^{3} \bar{F}_j d\left(\frac{a_{Vj}}{a_{jj}}\right) + \sum_{j=1}^{3} \frac{a_{Vj}}{a_{jj}} dF_j \tag{19}$$

Substituting equation (18) into equation (19),

$$d\bar{V} = \sum_{j=1}^{3} \bar{F}_j \ \sigma_j \frac{a_{Vj}}{a_{jj}} (\hat{R}_j - \hat{R}_V) + \sum_{j=1}^{3} \frac{a_{Vj}}{a_{jj}} dF_j \tag{20}$$

and by letting $\lambda_{Vj} = \frac{a_{Vj}\ X_j}{V}$, (fraction of mobile factor used by industry j), along with $F_j = a_{jj}\ X_j$, equation (20), which is one of the basic equations of change, can be rewritten as follows:

$$\hat{V} = \sum_{j=1}^{3} (\lambda_{Vj}\sigma_j\hat{R}_j - \lambda_{Vj}\sigma_j\hat{R}_V) + \sum_{j}^{3} \lambda_{Vj}\hat{F}_j \tag{21}$$

By differentiating equations (12), (13) and (14), three additional basic equations of change are formed:

$$\theta_{11}\ \hat{R}_1 + \theta_{V1}\ \hat{R}_V = \hat{P}_1^s - \theta_{11}\ \hat{a}_1 - \theta_{V1}\ \hat{a}_V \tag{22}$$

$$\theta_{22}\ \hat{R}_2 + \theta_{V2}\ \hat{R}_V = \hat{P}_2^s - \theta_{22}\ \hat{a}_2 - \theta_{V2}\ \hat{a}_V \tag{23}$$

$$\theta_{33}\ \hat{R}_3 + \theta_{V3}\ \hat{R}_V = \hat{P}_3^s - \theta_{33}\ \hat{a}_3 - \theta_{V3}\ \hat{a}_V \tag{24}$$

where $\theta_{ij} = \frac{a_{ij}\ R_i}{P_j^s}$ where $\left.\begin{array}{l} i = 1,\ 2,\ 3 = j \\ i = V \neq j \end{array}\right\}$ $\theta_{jj} + \theta_{Vj} = 1$

But for cost minimization $\theta_{jj}\ \hat{a}_j + \theta_{Vj}\ \hat{a}_V = 0$

Therefore, (22), (23) and (24) are in the form

$$\theta_{jj}\ \hat{R}_j + \theta_{Vj}\ \hat{R}_V = \hat{P}_j^s$$

Equations (21) to (24) can be expressed in matrix form as follows:

$$\begin{bmatrix} \theta_{11} & 0 & 0 & \theta_{V1} \\ 0 & \theta_{22} & 0 & \theta_{V2} \\ 0 & 0 & \theta_{33} & \theta_{V3} \\ \lambda_{V1}\sigma_1 & \lambda_{V2}\sigma_2 & \lambda_{V3}\sigma_3 & -\sum_{j=1}^{3} \lambda_{Vj}\sigma_j \end{bmatrix} \begin{bmatrix} \hat{R}_1 \\ \hat{R}_2 \\ \hat{R}_3 \\ \hat{R}_V \end{bmatrix} = \begin{bmatrix} \hat{P}_1^s \\ \hat{P}_2^s \\ \hat{P}_3^s \\ \hat{V} - \sum_{j=1}^{3} \lambda_{Vj}\hat{F}_j \end{bmatrix} \tag{25}$$

Because we are drawing the variable factor out of other activities while leaving the fixed factor alone, then $\hat{F}_j = 0$.

Using Cramer's Rule to solve (25) for $\hat{R}_j$ and $\hat{R}_V$ we have[1]:

$$\hat{R}_1 = \frac{1}{\text{Det.}} \begin{vmatrix} \hat{P}_1^s & 0 & 0 & \theta_{V1} \\ \hat{P}_2^s & \theta_{22} & 0 & \theta_{V2} \\ \hat{P}_3^s & 0 & \theta_{33} & \theta_{V3} \\ \hat{V} & \lambda_{V2}\,\sigma_2 & \lambda_{V3}\,\sigma_3 & -\sum_{j=1}^{3} \lambda_{Vj}\,\sigma_j \end{vmatrix}$$

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1. Solution of the determinant of the coefficient matrix of (25) is as follows:

$$\text{Det.} = \begin{vmatrix} \theta_{11} & 0 & 0 & \theta_{V1} \\ 0 & \theta_{22} & 0 & \theta_{V2} \\ 0 & 0 & \theta_{33} & \theta_{V3} \\ \lambda_{V1}\sigma_1 & \lambda_{V2}\sigma_2 & \lambda_{V3}\sigma_3 & -\sum_{j=1}^{3} \lambda_{Vj}\sigma_j \end{vmatrix}$$

$$\text{Det.} = -\lambda_{V1}\sigma_1(\theta_{22}\theta_{33}\theta_{V1}) + \lambda_{V2}\sigma_2(-\theta_{11}\theta_{33}\theta_{V2}) - \lambda_{V3}\sigma_3(\theta_{11}\theta_{22}\theta_{V3}) - \sum_{j=1}^{3} \lambda_{Vj}\sigma_j(\theta_{11}\theta_{22}\theta_{33})$$

$$\delta = \frac{(-1)\,\text{Det.}}{\theta_{11}\theta_{22}\theta_{33}} = (\theta_{V1}\lambda_{V1}\frac{\sigma_1}{\theta_{11}} + \theta_{V2}\lambda_{V2}\frac{\sigma_2}{\theta_{22}} + \theta_{V3}\lambda_{V3}\frac{\sigma_3}{\theta_{33}} + \sum_{j=1}^{3} \lambda_{Vj}\sigma_j)$$

$$= \left[\lambda_{V1}\sigma_1(1 + \frac{\theta_{V1}}{\theta_{11}}) + \lambda_{V2}\sigma_2(1 + \frac{\theta_{V2}}{\theta_{22}}) + \lambda_{V3}\sigma_3(1 + \frac{\theta_{V3}}{\theta_{33}})\right]$$

$$= \left[\lambda_{V1}\frac{\sigma_1}{\theta_{11}} + \lambda_{V2}\frac{\sigma_2}{\theta_{22}} + \lambda_{V3}\frac{\sigma_3}{\theta_{33}}\right]$$

$$\hat{R}_1 = \frac{1}{Det.} \Big[\hat{P}_1^s \left(-\theta_{22}\theta_{33} \sum_{j=1}^{3} \lambda_{Vj}\sigma_j - \theta_{33}\theta_{V2}\lambda_{V2}\sigma_2 - \theta_{22}\theta_{V3}\lambda_{V3}\sigma_3\right)$$

$$- \hat{P}_2^s \left(-\theta_{33}\theta_{V1}\lambda_{V2}\sigma_2\right) + \hat{P}_3^s \left(\theta_{22}\theta_{V1}\lambda_{V3}\sigma_3\right) - \hat{V}\left(\theta_{22}\theta_{33}\theta_{V1}\right) \Big]$$

$$\hat{R}_1 = \frac{1}{\delta} \Big[+ \left(\sum_{j=1}^{3} \lambda_{Vj} \frac{\sigma_j}{\theta_{11}} + \frac{\theta_{V2}}{\theta_{11}} \lambda_{V2} \frac{\sigma_2}{\theta_{22}} + \frac{\theta_{V3}}{\theta_{11}} \lambda_{V3} \frac{\sigma_3}{\theta_{33}}\right) \hat{P}_1^s$$

$$- \left(\frac{\theta_{V1}}{\theta_{11}} \lambda_{V2} \frac{\sigma_2}{\theta_{22}}\right) \hat{P}_2^s - \left(\frac{\theta_{V1}}{\theta_{11}} \lambda_{V3} \frac{\sigma_3}{\theta_{33}}\right) \hat{P}_3^s + \left(\frac{\theta_{V1}}{\theta_{11}}\right) \hat{V} \Big] \tag{26}$$

$$\hat{R}_2 = \frac{1}{Det.} \begin{vmatrix} \theta_{11} & \hat{P}_1^s & 0 & \theta_{V1} \\ 0 & \hat{P}_2^s & 0 & \theta_{V2} \\ 0 & \hat{P}_3^s & \theta_{33} & \theta_{V3} \\ \lambda_{V1}\sigma_1 & \hat{V} & \lambda_{V3}\sigma_3 & -\sum_{j=1}^{3} \lambda_{Vj}\sigma_j \end{vmatrix}$$

$$\hat{R}_2 = \frac{1}{Det.} \Big[- \hat{P}_1^s \left(-\theta_{33}\theta_{V2}\lambda_{V1}\sigma_1\right)$$

$$+ \hat{P}_2^s \left(-\theta_{11}\theta_{33} \sum_{j=1}^{3} \lambda_{Vj}\sigma_j - \theta_{33}\theta_{V1}\lambda_{V1}\sigma_1 - \theta_{11}\theta_{V3}\lambda_{V3}\sigma_3\right)$$

$$- \hat{P}_3^s \left(-\theta_{11}\theta_{V2}\lambda_{V3}\sigma_3\right)$$

$$+ \hat{V} \left(-\theta_{11}\theta_{33}\theta_{V2}\right) \Big]$$

$$\hat{R}_2 = \frac{1}{\delta}\Bigg[- \left(\frac{\theta_{V2}}{\theta_{22}}\,\lambda_{V1}\,\frac{\sigma_1}{\theta_{11}}\right)\hat{P}^s_1$$

$$+ \left(\sum_{j=1}^{3}\lambda_{Vj}\,\frac{\sigma_j}{\theta_{22}} + \frac{\theta_{V1}}{\theta_{22}}\,\lambda_{V1}\,\frac{\sigma_1}{\theta_{11}} + \frac{\theta_{V3}}{\theta_{22}}\,\lambda_{V3}\,\frac{\sigma_3}{\theta_{33}}\right)\hat{P}^s_2 \tag{27}$$

$$- \left(\frac{\theta_{V2}}{\theta_{22}}\,\lambda_{V3}\,\frac{\sigma_3}{\theta_{33}}\right)\hat{P}^s_3 + \left(\frac{\theta_{V2}}{\theta_{22}}\right)\hat{V}\Bigg]$$

$$\hat{R}_3 = \frac{1}{\text{Det.}}\begin{vmatrix} \theta_{11} & 0 & \hat{P}^s_1 & \theta_{V1} \\ 0 & \theta_{22} & \hat{P}^s_2 & \theta_{V2} \\ 0 & 0 & \hat{P}^s_3 & \theta_{V3} \\ \lambda_{V1}\sigma_1 & \lambda_{V2}\sigma_2 & \hat{V} & -\sum_{j=1}^{3}\lambda_{Vj}\sigma_j \end{vmatrix}$$

$$\hat{R}_3 = \frac{1}{\text{Det.}}\Bigg[+ \hat{P}^s_1\,(\theta_{22}\theta_{V3}\lambda_{V1}\sigma_1) - \hat{P}^s_2\,(-\theta_{11}\theta_{V3}\lambda_{V2}\sigma_2)$$

$$+ \hat{P}^s_3\,(-\theta_{11}\theta_{22}\sum_{j=1}^{3}\lambda_{Vj}\sigma_j - \theta_{22}\theta_{V1}\lambda_{V1}\sigma_1 - \theta_{11}\theta_{V2}\lambda_{V2}\sigma_2)$$

$$- \hat{V}\,(\theta_{11}\theta_{22}\theta_{V3})\Bigg]$$

$$\hat{R}_3 = \frac{1}{\delta}\Big[-\left(\frac{\theta_{V3}}{\theta_{33}}\lambda_{V1}\frac{\sigma_1}{\theta_{11}}\right)\hat{P}^s_1 - \left(\frac{\theta_{V3}}{\theta_{33}}\lambda_{V2}\frac{\sigma_2}{\theta_{22}}\right)\hat{P}^s_2$$

$$+ \left(\sum_{j=1}^{3}\lambda_{Vj}\frac{\sigma_j}{\theta_{33}} + \frac{\theta_{V1}}{\theta_{33}}\lambda_{V1}\frac{\sigma_1}{\theta_{11}} + \frac{\theta_{V2}}{\theta_{33}}\lambda_{V2}\frac{\sigma_2}{\theta_{22}}\right)\hat{P}^s_3$$

$$+ \left(\frac{\theta_{V3}}{\theta_{33}}\right)\hat{V}\Big] \tag{28}$$

$$\hat{R}_V = \frac{1}{\text{Det.}}\begin{vmatrix} \theta_{11} & 0 & 0 & \hat{P}^s_1 \\ 0 & \theta_{22} & 0 & \hat{P}^s_2 \\ 0 & 0 & \theta_{33} & \hat{P}^s_3 \\ \lambda_{V1}\sigma_1 & \lambda_{V2}\sigma_2 & \lambda_{V3}\sigma_3 & \hat{V} \end{vmatrix}$$

$$= \frac{1}{\text{Det.}}\Big[-\hat{P}^s_1(\theta_{22}\theta_{33}\lambda_{V1}\sigma_1) + \hat{P}^s_2(-\theta_{11}\theta_{33}\lambda_{V2}\sigma_2)$$

$$- \hat{P}^s_3(\theta_{11}\theta_{22}\lambda_{V3}\sigma_3) + \hat{V}(\theta_{11}\theta_{22}\theta_{33})\Big]$$

$$= \frac{1}{\delta}\Big[+\left(\lambda_{V1}\frac{\sigma_1}{\theta_{11}}\right)\hat{P}^s_1 + \left(\lambda_{V2}\frac{\sigma_2}{\theta_{22}}\right)\hat{P}^s_2 \tag{29}$$

$$+ \left(\lambda_{V3}\frac{\sigma_3}{\theta_{11}}\right)\hat{P}^s_3 - \hat{V}\Big]$$

where δ = (-1) times determinant of the coefficient matrix divided by $(\theta_{11}\theta_{22}\theta_{33})$

$$= \left[\lambda_{V1}\frac{\sigma_1}{\theta_{11}} + \lambda_{V2}\frac{\sigma_2}{\theta_{22}} + \lambda_{V3}\frac{\sigma_3}{\theta_{33}}\right]$$

$\frac{\sigma_j}{\theta_{Vj}}$ = inverse of the elasticity of the marginal physical product of the variable factor (V) in sector j.

We need to know the reaction of output in each sector to a change in relative prices as a result of withdrawing 1 unit of the variable factor for the project.

Total differentiation of equations (8), (9) and (10) yields

$$\hat{X}_1^s = -\hat{a}_{11} \tag{30}$$

$$\hat{X}_2^s = -\hat{a}_{22} \tag{31}$$

$$\hat{X}_3^s = -\hat{a}_{33} \tag{32}$$

when $\hat{F}_j = 0$

From cost minimization

$$\theta_{Vj}\hat{a}_{Vj} + \theta_{jj}\hat{a}_{jj} = 0$$

$$\therefore \hat{a}_{Vj} = -\frac{\theta_{jj}}{\theta_{Vj}}\hat{a}_{jj}$$

Substitute into (18)[1]

$$\hat{a}_{jj} - \hat{a}_{Vj} = - \sigma_j \; (\hat{R}_j - \hat{R}_V)$$

$$\hat{a}_{jj} \; (1 + \frac{\theta_{jj}}{\theta_{Vj}}) = - \sigma_j \; (\hat{R}_j - \hat{R}_V)$$

$$\hat{a}_{jj} = - \theta_{Vj} \; \sigma_j \; (\hat{R}_j - \hat{R}_V) \tag{33}$$

Substituting equation (33) into equation (30) and then substituting equations (26) to (29) for $\hat{R}_1$ and $\hat{R}_V$ we obtain an expression for the determination of $\hat{X}_1^s$ as follows:

$$\hat{X}_1^s = \theta_{Vj} \; \sigma_j \; (\hat{R}_1 - \hat{R}_V) \text{ or}$$

$$\begin{aligned}
\hat{X}_1^s = \frac{\theta_{V1}\sigma_1}{\theta_{11}\;\delta} \Big[& (\lambda_{V2}\sigma_2 + \lambda_{V3}\sigma_3 + \theta_{V2}\lambda_{V2}\frac{\sigma_2}{\theta_{22}} + \theta_{V3}\lambda_{V3}\frac{\sigma_3}{\theta_{33}}) \; \hat{P}_1^s \\
& - (\theta_{V1}\lambda_{V2}\frac{\sigma_2}{\theta_{22}} + \theta_{11}\lambda_{V2}\frac{\sigma_2}{\theta_{22}}) \; \hat{P}_2^s \\
& - (\theta_{V1}\lambda_{V3}\frac{\sigma_3}{\theta_{33}} + \theta_{11}\lambda_{V3}\frac{\sigma_3}{\theta_{33}}) \; \hat{P}_3^s \\
& + (\theta_{V1} + \theta_{11}) \; \hat{V} \Big]
\end{aligned}$$

$$\begin{aligned}
\hat{X}_1^s = \frac{\theta_{V1}\sigma_1}{\delta\;\theta_{11}} \Big[& (\lambda_{V2}\frac{\sigma_2}{\theta_{22}} + \lambda_{V3}\frac{\sigma_3}{\theta_{33}}) \; \hat{P}_1^s - (\lambda_{V2}\frac{\sigma_2}{\theta_{22}}) \; \hat{P}_2^s \\
& - (\lambda_{V3}\frac{\sigma_3}{\theta_{33}}) \; \hat{P}_3^s + \hat{V} \Big]
\end{aligned} \tag{34}$$

Similarly for $\hat{X}_2^s$ and $\hat{X}_3^s$,

$$\hat{X}_2^s = \frac{\theta_{V2}}{\delta} \frac{\sigma_2}{\theta_{22}} \Big[- (\lambda_{V1} \frac{\sigma_1}{\theta_{11}}) \hat{P}_1^s + (\lambda_{V1} \frac{\sigma_1}{\theta_{11}} + \lambda_{V3} \frac{\sigma_3}{\theta_{33}}) \hat{P}_2^s - (\lambda_{V3} \frac{\sigma_3}{\theta_{33}}) \hat{P}_3^s + \hat{V} \Big] \qquad (35)$$

$$\hat{X}_3^s = \frac{\theta_{V3}}{\delta} \frac{\sigma_3}{\theta_{33}} \Big[- (\lambda_{V1} \frac{\sigma_1}{\theta_{11}}) \hat{P}_1^s - (\lambda_{V2} \frac{\sigma_2}{\theta_{22}}) \hat{P}_2^s + (\lambda_{V2} \frac{\sigma_2}{\theta_{22}} + \lambda_{V3} \frac{\sigma_3}{\theta_{33}}) \hat{P}_3^s + \hat{V} \Big] \qquad (36)$$

From the supply equations (34) to (36), we find that a change in relative prices of the outputs will create a movement of the variable factor between sectors until its return is again equalized. So far in describing this model the output prices have been assumed to be exogeneous. However, because the world prices of the traded goods are determined exogeneously, the domestic prices of these traded goods will vary directly with the exchange rate. The exchange rate in turn is determined by the demand and supply of foreign exchange. In the case of the non-traded good, its price will be determined by the interaction of the domestic demand and supply of these goods.

The change in the domestic demand for each of the three goods can be written as the sum of the (substitution effect only) own and cross price elasticities of demand times their corresponding changes in prices, plus the income elasticity of demand for the good multiplied by the change in real income, plus the change in the demand for the good by government. The domestic market demand function for the importable, exportable and non-traded goods are presented in equations (37) to (39).

Importable Good Demand $$\hat{X}_1^d = N_{11}\hat{D}_1 + N_{12}\hat{D}_2 + N_{13}\hat{D}_3 + N_{1y}\frac{dy}{y} \qquad (37)$$

Exportable Good Demand $$\hat{X}_2^d = N_{21}\hat{D}_1 + N_{22}\hat{D}_2 + N_{23}\hat{D}_3 + N_{2y}\frac{dy}{y} \qquad (38)$$

Non-traded Good Demand $$\hat{X}_3^d = N_{31}\hat{D}_1 + N_{32}\hat{D}_2 + N_{33}\hat{D}_3 + N_{3y}\frac{dy}{y} \qquad (39)$$

where:

N_{ij} is the substitution effect only demand elasticity of good i with respect to the price of good j

i.e. $\left(\frac{\partial X_i^d}{\partial D_j} \frac{D_j}{X_i^d} \right)$

N_{iy} is the elasticity of demand for good i with respect to real income

i.e. $\left(\frac{\partial X_i^d}{\partial y} \frac{y}{X_i^d} \right)$

While changes in domestic demand for importable type goods are described by equation (37) the demand for imports (M) is described by the excess of the domestic demand for importable type goods over the domestic supply.

$$M = X_1^d - X_1^s \tag{40}$$

when expressed as changes in quantities equation (40) becomes,

$$\hat{M} = \frac{X_1^d}{M}\hat{X}_1^d - \frac{X_1^s}{M}\hat{X}_1^s \tag{41}$$

Substituting the equation for the change in domestic demand for importables (37) and the equation for the domestic supply of importable (34) into equation (41), and choosing the price of the non-traded good as the numeraire, i.e., (set $\hat{P}_3^s = 0$), we obtain the following expression for the change in the demand for imports.[1]

$$\begin{aligned}\hat{M} = {} & \left[\frac{X_1^d}{M}\eta_{11} - \frac{X_1^s}{M}\left(\frac{\lambda_{V2}\theta_{V1}}{\delta}\frac{\sigma_1\ \sigma_2}{\theta_{11}\theta_{22}} + \frac{\lambda_{V3}\theta_{V1}}{\delta}\frac{\sigma_1\ \sigma_3}{\theta_{11}\theta_{33}}\right)\right]\hat{P}_1^s \\ & + \left[\frac{X_1^d}{M}\eta_{12} + \frac{X_1^s}{M}\frac{\lambda_{V2}\theta_{V1}}{\delta}\frac{\sigma_1\ \sigma_2}{\theta_{11}\theta_{22}}\right]\hat{P}_2^s + \left[\frac{X_1^d}{M}\eta_{1y}\right]\hat{Y} \\ & - \left[\frac{X_1^s}{M}\frac{\theta_{V1}}{\delta}\frac{\sigma_1}{\theta_{11}}\right]\hat{V}\end{aligned} \tag{42}$$

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1. Because we assume the ad valorem tax, tariff and subsidy rates are held constant therefore,

$$\hat{P}_1^s = \hat{D}_1,\ \hat{P}_2^s = \hat{D}_2,\ \hat{P}_3^s = \hat{D}_3.$$

In a similar fashion, the supply of exports (E) is equal to the total domestic supply of the exportable type good less the domestic demand for the good.

$$E = X_2^s - X_2^d \tag{43}$$

Expressed in terms of changes in exports supplied:

$$\hat{E} = \frac{X_2^s}{E}\hat{X}_2^s - \frac{X_2^d}{E}\hat{X}_2^d \tag{44}$$

Substituting equations (35) and (38) into equation (44) and setting $\hat{D}_3 = 0$, we have:

$$\hat{E} = -\left[\frac{X_2^s}{E}\frac{\lambda_{V1}\theta_{V2}}{\delta}\frac{\sigma_1\ \sigma_2}{\theta_{11}\theta_{22}} + \frac{X_2^d}{E}\eta_{21}\right]\hat{P}_1^s + \left[\frac{X_2^s}{E}\frac{\lambda_{V1}\theta_{V2}}{\delta}\frac{\sigma_1\sigma_2}{\theta_{11}\theta_{22}}\right.$$
$$\left. + \frac{X_2^s}{E}\frac{\lambda_{V3}\theta_{V2}}{\delta}\frac{\sigma_2\ \sigma_3}{\theta_{22}\theta_{33}} - \frac{X_2^d}{E}\eta_{22}\right]\hat{P}_2^s - \left[\frac{X_2^d}{E}\eta_{2y}\right]\hat{y} + \left[\frac{X_2^s}{E}\frac{\theta_{V2}}{\delta}\frac{\sigma_2}{\theta_{22}}\right]\hat{V} \tag{45}$$

As all the goods are initially measured in units of foreign exchange and the ad valorem rates of taxes, tariffs and subsidies are assumed to be held constant, the change in the producer's price of importables ($\hat{P}_1^s$) arising from a change in governments demand for imports or a change in real income will be equal to the change in the producers price of exports ($\hat{P}_2^s$), which will also be equal to the change in the market price of foreign exchange ($\hat{E}_m$). If initially the market for foreign exchange was in equilibrium and remains in such a state, i.e. M = E, it will require that $\hat{M} = \hat{E}$. By equating equations (42) and (45) and simplifying we obtain an expression which describes the changes in the market exchange rate if none of the domestic and trade distortions are altered.

$$\hat{E}_m = \frac{\left[X_1^d \eta_{1y} + X_2^d \eta_{2y}\right]\hat{y} - \left[X_1^s \frac{\theta_{V1}\ \sigma_1}{\delta\ \theta_{11}} + X_2^s \frac{\theta_{V2}\ \sigma_2}{\delta\ \theta_{22}}\right]\hat{V}}{X_1^s \frac{\lambda_{V3}\theta_{V1}}{\delta}\frac{\sigma_1\ \sigma_3}{\theta_{11}\theta_{33}} + X_2^s \frac{\lambda_{V2}\theta_{V2}}{\delta}\frac{\sigma_2\ \sigma_3}{\theta_{22}\theta_{33}} - (\eta_{11}+\eta_{12})X_1^d - (\eta_{21}+\eta_{22})X_2^d}$$

$$\hat{E}_m = \hat{P}_1^s = \hat{P}_2^s = \hat{D}_1 = \hat{D}_2$$

$$\hat{P}_3 = \hat{D}_3 = 0$$

As the quantities of all goods are defined in terms of units of foreign exchange, the value added of labour V can also be measured in units of foreign exchange. If we arbitrarily choose units of good 3 such that $P_3^s = 1$, then the initial market exchange rate (E_m) will also be equal to one. The units of V can also be chosen so that at the initial equilibrium the price of V = 1.

In order to obtain a unit (i.e. a dollar's worth at understated prices) of the variable factor dV holding constant real disposable income in the economy, we have $\hat{y} = 0$ and dV = -1. Therefore:

$$\hat{E}_m = \frac{-\left[X_1^s \frac{\theta_{V1}\ \sigma_1}{\delta\ \theta_{11}} + X_2^s \frac{\theta_{V2}\ \sigma_2}{\delta\ \theta_{22}}\right]\frac{dV}{V}}{A}$$

Since $\lambda_{Vj} = a_{Vj}\frac{X_j}{V}$

$$\frac{X_1^s}{V} = \frac{\lambda_{V1}}{a_{V1}} \quad \text{and} \quad \frac{X_2^s}{V} = \frac{\lambda_{V2}}{a_{V2}}$$

$$\hat{E}_m = \frac{-\left[\frac{\lambda_{V1}}{a_{V1}} \frac{\theta_{V1}\sigma_1}{\delta\ \theta_{11}} + \frac{\lambda_{V2}}{a_{V2}} \frac{\theta_{V2}}{\delta} \frac{\sigma_2}{\theta_{22}}\right] dV}{A} \tag{46}$$

$$\text{where } A = X_1^s \frac{\lambda_{V3}\theta_{V1}}{\delta} \frac{\sigma_1\ \sigma_3}{\theta_{11}\theta_{33}} + X_2^s \frac{\lambda_{V3}\theta_{V2}}{\delta} \frac{\sigma_2\ \sigma_3}{\theta_{22}\theta_{33}} - (\eta_{11}+\eta_{12})X_1^d$$

$$- (\eta_{21}+\eta_{22})\ X_2^d$$

To measure the gross-of-sales-tax-and-trade-distortion cost of the labour now used in the project we must first evaluate the net change in domestic supply and domestic demand of each of the goods as a result of the reduction in the variable factor available to produce these goods. These changes in the quantities supplied and demanded of each of the goods will then be evaluated according to their value in consumption or their resource cost of production. This measurement of the value of the goods foregone to extract an additional unit of labour is carried out using prices and resource costs that existed prior to the project's inception.

(a) <u>Change in Private Demand for Importables</u>

Because $\hat{E}_m = \hat{P}_1^s = \hat{P}_2^s = \hat{D}_1 = \hat{D}_2$, and $\hat{P}_3 = \hat{D}_3 = 0$, and $dy = 0$, we can substitute equation (46) into (37) and solve for the change in <u>private</u> demand for importables. From equation (1) the private consumers evaluate a unit of consumption of the importable good at the demand price which is equal to $E_m(1+\tau_1)(1+t_1)$. Therefore, their evaluation of the change in the demand for the importable good can be expressed as follows:

Value of Change in Private Demand for Importables =

$$E_m(1+\tau_1)(1+t_1)\, d\hat{X}_1^d \;=\; X_1^d\,(\eta_{11}+\eta_{12})\,\hat{E}_m\left[E_m(1+\tau_1)(1+t_1)\right]$$

$$= \frac{X_1^d(\eta_{11}+\eta_{12})}{A}\left[\frac{\lambda_{V1}}{a_{V1}}\,\frac{\theta_{V1}}{\delta}\,\frac{\sigma_1}{\theta_{11}} + \frac{\lambda_{V2}}{a_{V2}}\,\frac{\theta_{V2}}{\delta}\,\frac{\sigma_2}{\theta_{22}}\right]\left[E_m(1+\tau_1)(1+t_1)\right] \tag{47}$$

where dV = -1

(b) Change in Private Demand for Exportables

From equation 3 we know consumers value an additional unit of consumption of the exportable good by an amount equal to its demand price of $E_m(1+t_2)$. Substituting equation (46) into equation (38) and solving:

$$E_m(1+t_2)\,d\hat{X}_2^d \;=\; X_2^d(\eta_{21}+\eta_{22})\hat{E}_m\left[E_m(1+t_2)\right]$$

$$= \frac{X_2^d\;(\eta_{21}+\eta_{22})}{A}\left[\frac{\lambda_{V1}}{a_{V1}}\,\frac{\theta_{V1}}{\delta}\,\frac{\sigma_1}{\theta_{11}} + \frac{\lambda_{V2}}{a_{V2}}\,\frac{\theta_{V2}}{\delta}\,\frac{\sigma_2}{\theta_{22}}\right]\left[E_m(1+t_2)\right] \tag{48}$$

where dV = -1

(c) Change in Private Demand for Non-Traded Goods

Consumers value an additional unit of consumption of the non-traded good by an amount equal to its demand price of $E_m(1+t_3)$. Substituting equation (46) into equation (39) and using the relationship for the substitution effect only price elasticities of demand that $N_{31} + N_{32} + N_{33} = 0$, we obtain:

$$E_m(1+t_3)\,dX_3^d = X_3^d(-\eta_{33})\ \hat{E}_m\ \left[E_m(1+t_3)\right]$$

$$= \frac{-X_3^d(\eta_{33})}{A}\left[\frac{\lambda_{V1}}{a_{V1}}\ \frac{\theta_{V1}}{\delta}\ \frac{\sigma_1}{\theta_{11}} + \frac{\lambda_{V2}}{a_{V2}}\ \frac{\theta_{V2}}{\delta}\ \frac{\sigma_2}{\theta_{22}}\right]\left[E_m(1+t_3)\right] \qquad (49)$$

where $dV = -1$

(d) <u>Change in Domestic Supply of Importable Good</u>

The resource cost of additional domestic supplies of the importable good will be equal to the foreign exchange cost of imports gross of the tariff, i.e. $E_m(1+\tau_1)$. From equations (46) and (34) we obtain the resource cost of additional domestic supplies of importables brought about by this action.

Value of change in resources used to produce importables =

$$E_m(1+\tau_1)\,dX_1^s = X_1^s\left[\left(\frac{\theta_{V1}\lambda_{V3}}{\delta}\ \frac{\sigma_1\ \ \sigma_3}{\theta_{11}\theta_{33}}\right)\hat{E}_m + \frac{\theta_{V1}\sigma_1}{\delta\ \theta_{11}}\ \hat{V}\right]\left[E_m(1+\tau_1)\right]$$

$$= X_1^s\left[\frac{\theta_{V1}\ \sigma_1}{\delta\ \theta_{11}} - \frac{\theta_{V1}\lambda_{V3}}{\lambda}\ \frac{\sigma_1}{\theta_{11}}\ \frac{\sigma_3}{\theta_{33}}\ \frac{\left(X_1^s\frac{\theta_{V1}\sigma_1}{\delta\ \theta_{11}} + X_2^s\frac{\theta_{V2}\ \sigma_2}{\delta\ \theta_{22}}\right)}{A}\right]\hat{V}\ \left[E_m(1+\tau_1)\right]$$

$$= \left[\frac{\lambda_{V1}}{a_{V1}}\ \frac{\theta_{V1}\sigma_1}{\delta\ \theta_{11}} + X_1^s\theta_{V1}\ \frac{\lambda_{V3}}{\delta}\ \frac{\sigma_1\ \sigma_3}{\theta_{11}\theta_{33}}\ \frac{\left(\frac{\lambda_{V1}}{a_{V1}}\ \frac{\theta_{V1}}{\delta}\ \frac{\sigma_1}{\theta_{11}} + \frac{\lambda_{V2}}{a_{V2}}\ \frac{\theta_{V2}}{\delta}\ \frac{\sigma_2}{\theta_{22}}\right)}{A}\right]E_m(1+\tau_1 \qquad (50)$$

where $\frac{X_j^s}{V} = \frac{\lambda_{Vj}}{a_{Vj}}$ and $dV = -1$

(e) <u>Change in Domestic Supply of Exportable Good</u>

The resource cost of this sector of an additional unit of domestic production is equal to $E_m(1+k_2)$. From equations (46) and (35) we have:

Value of change in resources used to produce exportables =

$$E_m(1+k_2)dX_2^s = X_2^s\left[\left(\frac{\theta_{V2}\lambda_{V3}}{\delta}\frac{\sigma_2\ \sigma_3}{\theta_{22}\theta_{33}}\right)\hat{E}_m + \frac{\theta_{V2}\ \sigma_2}{\delta\ \theta_{22}}\hat{V}\right]\left[E_m(1+k_2)\right]$$

$$= \left[X_2^s\ \theta_{V2}\ \frac{\lambda_{V3}}{\delta}\ \frac{\sigma_2}{\theta_{22}}\ \frac{\sigma_3}{\theta_{33}}\ \frac{\frac{\lambda_{V1}}{a_{V1}}\ \frac{\theta_{V1}}{\delta}\ \frac{\sigma_1}{\theta_{11}} + \frac{\lambda_{V2}}{a_{V2}}\ \frac{\theta_{V2}}{\delta}\ \frac{\sigma_2}{\theta_{22}})}{A} - \frac{\lambda_{V2}}{a_{V2}}\ \frac{\theta_{V2}}{\delta}\ \frac{\sigma_2}{\theta_{22}}\right]E_m(1+k_2) \tag{51}$$

(f) <u>Change in Domestic Supply of Non-Traded Good</u>

Initially the resource cost of production of an additional unit of the non-traded good is equal to E_m. From equations (46) and (36) we have:

$$E_m dX_3^s = X_3^s\left[-\left(\theta_{V3}\ \frac{\lambda_{V1}}{\delta}\ \frac{\sigma_1}{\theta_{11}}\ \frac{\sigma_3}{\theta_{33}} + \theta_{V3}\ \frac{\lambda_{V1}}{\delta}\ \frac{\sigma_2}{\theta_{22}}\ \frac{\sigma_3}{\theta_{33}}\right)\hat{E}_m + \frac{\theta_{V3}\sigma_3}{\delta\ \theta_{33}}\ \hat{V}\right]\ E_m$$

$$= -\left[X_3^s\ \left(\theta_{V3}\ \frac{\lambda_{V1}}{\delta}\ \frac{\sigma_1}{\theta_{11}}\ \frac{\sigma_3}{\theta_{33}} + \theta_{V3}\ \frac{\lambda_{V2}}{\delta}\ \frac{\sigma_2}{\theta_{22}}\ \frac{\sigma_3}{\theta_{33}}\right)\ \frac{\left(\frac{\lambda_{V1}}{a_{V1}}\ \frac{\theta_{V1}}{\delta}\ \frac{\sigma_1}{\theta_{11}} + \frac{\lambda_{V2}}{a_{V2}}\ \frac{\theta_{V2}}{\delta}\ \frac{\sigma_2}{\theta_{22}}\right)}{A}\right.$$

$$\left. + \frac{\lambda_{V3}}{a_{V3}}\ \frac{\theta_{V3}}{\delta}\ \frac{\sigma_3}{\theta_{33}}\right]E_m \tag{52}$$

where $dV = -1$

The total social cost of using one unit of the variable factor on a project is found by adding the value of resources used or released in production and the value of increased or decreased consumption. Equations (47) to (52) are combined to yield the social cost of the variable factor.

$$\text{Let } B = \frac{\lambda_{V1}}{a_{V1}} \frac{\theta_{V1}}{\delta} \frac{\sigma_1}{\theta_{11}}$$

$$C = \frac{\lambda_{V2}}{a_{V2}} \frac{\theta_{V2}}{\delta} \frac{\sigma_2}{\theta_{22}}$$

$$F = \theta_{V1} \frac{\lambda_{V3}}{\delta} \frac{\sigma_1}{\theta_{11}} \frac{\sigma_3}{\theta_{33}}$$

$$G = \theta_{V2} \frac{\lambda_{V3}}{\delta} \frac{\sigma_2}{\theta_{22}} \frac{\sigma_3}{\theta_{33}}$$

$$H = \frac{\lambda_{V3}}{a_{V3}} \frac{\theta_{V3}}{\delta} \frac{\sigma_3}{\theta_{33}}$$

Then the social cost of 1 unit of the variable factor equals:

$$= X_1^d\ (\eta_{11}+\eta_{12})(\frac{B+C}{A})\left[E_m(1+\tau_1)(1+t_1)\right] + X_2^d(\eta_{21}+\eta_{22})(\frac{B+C}{A})\left[E_m(1+t_2)\right]$$

$$- X_3^d\ (\eta_{33})(\frac{B+C}{A})\left[E_m(1+t_3)\right] + \left[X_1^s\ F(\frac{B+C}{A}) - B\right]\ E_m(1+\tau_1)$$

$$+ \left[X_2^s\ G(\frac{B+C}{A})-C\right] E_m(1+k_2) - \left[X_3^s(\frac{\theta_{V3}\lambda_{V1}}{\theta_{V1}\lambda_{V3}}\ F + \frac{\theta_{V3}\lambda_{V2}}{\theta_{V2}\lambda_{V3}}G)(\frac{B+C}{A}) + H\right] E_m$$

$$= E_m\left[(\frac{B+C}{A})\ \left[X_1^d(\eta_{11}+\eta_{12})(1+\tau_2)(1+t_1) + X_2^d(\eta_{21}+\eta_{22})(1+t_2)\right.\right.$$

$$- X_3^d(\eta_{33})(1+t_3) + X_1^s\ F(1+\tau_1) + X_2^s\ G(1+k_2)$$

$$\left.\left.- X_3^s\ (\frac{\theta_{V3}}{\theta_{V1}}\ \frac{\lambda_{V1}}{\lambda_{V3}}\ F + \frac{\theta_{V3}}{\theta_{V2}}\ \frac{\lambda_{V2}}{\lambda_{V3}}\ G)\right] - B(1+\tau_1) - C(1+k_2) - H\right] \qquad (54$$

Divide the numerator and denominator of the first term by y:
Social cost of dV

$$= E_m\left\{\frac{B+C}{A^1}\left[\frac{X_1^d}{y}(\eta_{11}+\eta_{12})(1+\tau_1)(1+t_1) + \frac{X_2^d}{y}\ (\eta_{21}+\eta_{22})(1+t_2)\right.\right.$$

$$- \frac{X_3^d}{y}(\eta_{33})(1+t_3) + \frac{X_1^s}{y}\ F(1+\tau_1) + \frac{X_2^s}{y}G(1+k_2)$$

$$\left.\left.- \frac{X_3^s}{y}(\frac{\theta_{V3}}{\theta_{V1}}\ \frac{\lambda_{V1}}{\lambda_{V3}}\ F + \frac{\theta_{V3}}{\theta_{V2}}\ \frac{\lambda_{V2}}{\lambda_{V3}}\ G)\right] - B(1+\tau_1) - C(1+k_2) - H\right\} \qquad (55)$$

where: $A^1 = \frac{X_1^s}{y}\ F + \frac{X_2^s}{y}\ G - \frac{X_1^d}{y}\ (\eta_{11}+\eta_{12}) - \frac{X_2^d}{y}\ (\eta_{21}+\eta_{22})$

Empirical Estimation

In order to estimate the above formula for the value of sales tax and foreign exchange for labour we first need to know the rate of tariff, subsidy or tax applicable to each of the sectors which will be influenced by this action of purchasing foreign exchange. In addition to the values of the distortions in each of the sectors, we require values for the weights that are applied to these distortions. These weights are functions of the elasticities of demand and elasticities of substitution in production for each of the sectors. In addition, estimates are required of factor shares by sector, the shares of production of each sector in gross national product, and the amount of each good consumed as a ratio of gross national product.

The values for these parameters have been estimated by Jenkins (1976) and are presented in Table 1 along with the solution of equation (55).

Conclusion:

In case 1 of Table 1 it is assumed that the elasticities of substitution between factor inputs in production are equal to .5. The solution to equation (55) for the amount of the sales tax and trade distortions indicate a value of 5.1 percent of the gross-of-income-tax component of the social opportunity cost of labour.

Case 2 uses the same values for the variables except for the assumptions concerning the elasticities of substitution, which are set equal to 1.0. The solution for this case gives as a value of the sales tax and trade distortions for a change in the demand for labour of 3.6 percent.

These two sets of assumptions concerning the elasticity of substitution cover the range of most of the empirical estimates of this value for Canada.[1] If an average is taken of these two cases, therefore, the value of this distortion amounts to approximately 4.5 percent of the gross of income tax wage component of the social opportunity cost of labour

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1. Kotowitz, Yehuda, "Capital Labour Substitution in Canadian Manufacturing 1926-39 and 1946-61", Canadian Journal of Economics, August, 1968.

Table 1

Estimates of the Value of a Unit of Labour Added Gross of Sales Taxes and Trade Distortions

Variable		Variable	
N_{1y}	1.0	θ_{11}	.46
N_{2y}	1.0	θ_{22}	.50
N_{3y}	1.0	θ_{33}	.30
N_{11}	-.8	θ_{V1}	.54
N_{12}	.2	θ_{V2}	.50
N_{22}	-.8	θ_{V3}	.70
N_{21}	.533	X_1^d/Y	.40
N_{33}	-.622	X_2^d/Y	.15
λ_{V1}	.14	X_3^d/Y	.45
λ_{V2}	.33	X_1^s/Y	.15
λ_{V3}	.53	X_2^s/Y	.40
		X_3^s/Y	.45
		τ_1	.1204
		t_1	.0727
		t_2	.1255
		t_3	.0314
		k_2	.0632

Final Solution of Equation (55)

Case 1 $\sigma_1 = \sigma_2 = \sigma_3 = 0.5$ Value of dV = 1.051

Case 2 $\sigma_1 = \sigma_2 = \sigma_3 = 1.0$ Value of dV = 1.036