

**COST-BENEFIT ANALYSIS FOR INVESTMENT DECISIONS,
CHAPTER 5:**

***SCALE, TIMING, LENGTH AND INTER-DEPENDENCIES
IN PROJECT SELECTION***

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ABSTRACT

It is generally agreed that a project's net present value (NPV) is the most important criterion for the financial and the economic evaluation of a project from either the owner's or economic perspective.. The NPV criterion requires that a project analyst recommend only projects with positive NPV. The next step is to endeavor to maximize the NPV. The reason for trying to maximize the NPV is to extract as much value from the project as possible. Ideally, we should strive to maximize the NPV of incremental net cash flows or net economic benefits. Of course, optimization cannot be pursued blindly; there may be repercussions for other stakeholders that need to be considered in the final decision making. There are other important considerations project analysts often encounter. These considerations include changes in project parameters like the scale of investment, the date of initiation of a project, the length of project life or interdependencies of project components. Each of them is addressed in this chapter by using the criterion of a project's net present value. This chapter explains how project analysts use the criterion of a project's net present value to make such decisions.

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CHAPTER 5

SCALE, TIMING, LENGTH AND INTER-DEPENDENCIES IN PROJECT SELECTION

5.1 Introduction

In the previous chapter, we have concluded that a project's net present value (NPV) is the most important criterion for the financial and the economic evaluation. The NPV criterion requires that a project analyst recommend only projects with positive NPV. The next step is to endeavor to maximize the NPV. The reason for trying to maximize the NPV is to extract as much value from the project as possible. Ideally, we should strive to maximize the NPV of incremental net cash flows or net economic benefits. Of course, optimization cannot be pursued blindly; there may be repercussions for other stakeholders that need to be considered in the final decision making.

There are other important considerations project analysts often encounter. These considerations include changes in project parameters like the scale of investment, the date of initiation of a project, the length of project life or interdependencies of project components. Each of them is addressed in this chapter by using the criterion of a project's net present value.

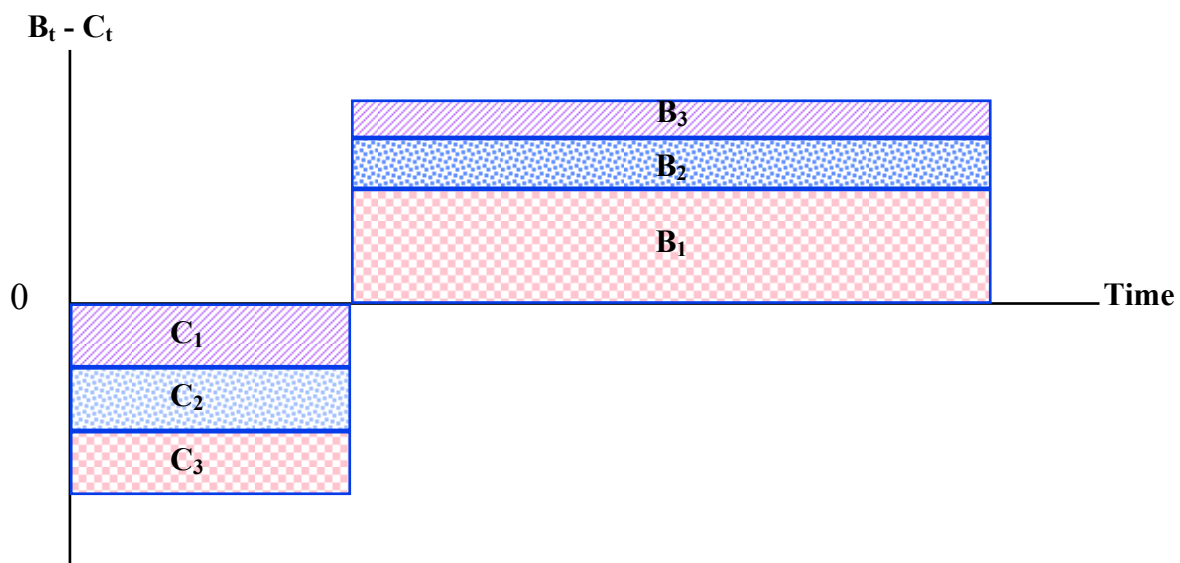
5.2 Determination of Scale in Project Selection

Projects are rarely, if ever, constrained by technological factors to a unique capacity or scale. Thus one of the most important decisions to be made in the design of a project is the selection of the scale at which a facility should be built. Far too often the scale selection has been treated as if it were a purely technical decision, neglecting its financial or economic aspects. When financial or economic considerations have been neglected at the design stage, the scale to which the project is built is not likely to be the one that would maximize the

NPV. Thus, in addition to technological factors the size of the market, the availability of project inputs, the quality of manpower, etc. will also have a role to play.

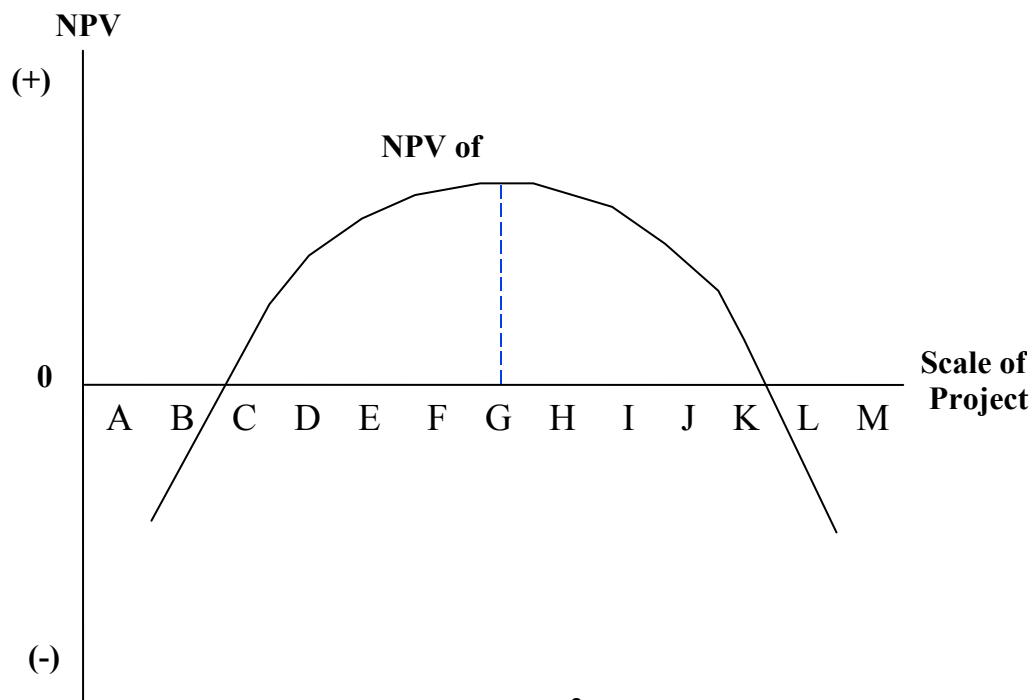
The most important principle for selection of the best scale of a project (e.g., height of an irrigation dam or size of a factory) is to treat each incremental change in its size as a project in itself. An increase in the scale of a project will require additional expenditures and will likely generate additional expected benefits over and above those that would have been produced by the project at its previous size. Using the present value of the incremental benefits and the present value of the incremental costs, the change in net present value, stemming from changing scales of the project, can be derived. In Figure 5.1 the cash flow profiles of a project are shown for three alternative scales. C_1 and B_1 denote the expected costs and benefits if the project is built at the smallest scale relevant for this evaluation. If the project is built at one size larger it will require additional expenditure of C_2 . Therefore, the total investment cost of the project at its expanded scale is $C_1 + C_2$. It is also anticipated that the benefits of the project will be increased by an amount of B_2 , implying that the total benefits from this scale of investment will now equal $B_1 + B_2$. A similar relationship holds for the largest scale of the project. In this case, additional expenditures of C_3 are required and extra benefits of B_3 are expected. Total investment costs for this scale equal $(C_1 + C_2 + C_3)$ and total benefits are $(B_1 + B_2 + B_3)$.

Figure 5.1 Net Benefit Profiles for Alternative Scales of a Facility



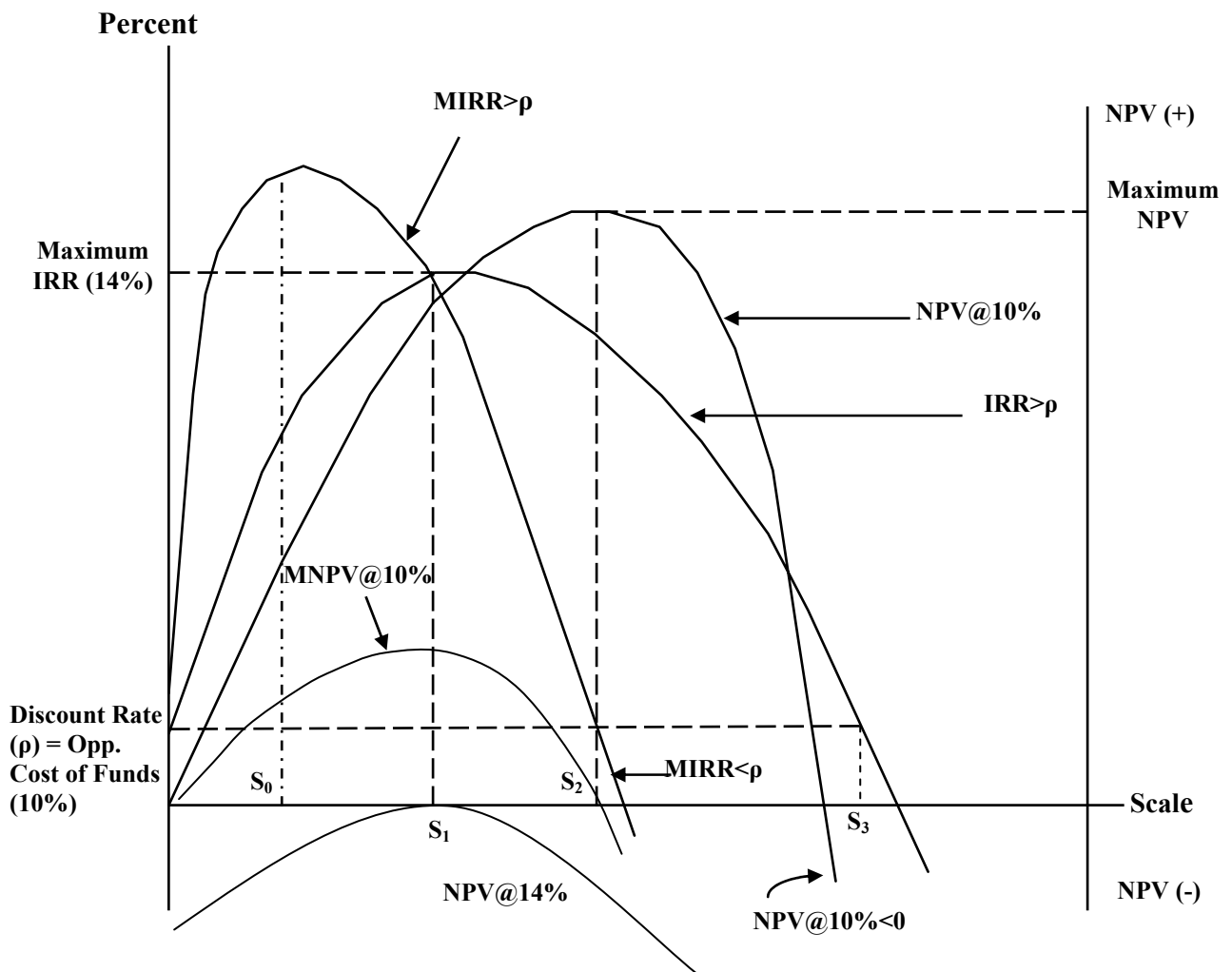
Our goal is to choose the scale that has the largest NPV. If the present value of $(B_1 - C_1)$ is positive, then it is a viable project. Next, we need to determine whether the present value of $(B_2 - C_2)$ is positive. If incremental NPV is positive, then this project at scale 2 is preferable to scale 1. This procedure is repeated until a scale is reached where the NPV of the incremental benefits and costs associated with a change in scale is negative. This incremental net present value approach helps us to choose a scale that has maximum NPV for the entire investment. The NPV is the maximum because the incremental NPV for any addition to the scale of the project would be negative. If the initial scale of the project had a negative NPV, but all the subsequent incremental net present values for changes of scale were positive, it still would be possible for the overall project to have a negative NPV. Therefore, in order to pick the optimum scale for a project, first we must make sure that the NPV of the overall project is positive and then the NPV of the last addition to the investment to increase project's scale must also be non-negative. This is illustrated in Figure 5.2 where all project sizes between scale C and scale M yield a positive NPV. However, the NPV of the entire investment is maximized at scale H. After scale H, the incremental NPV of any expansion of the facility becomes negative. Therefore, the optimum scale for the project is H, even though the NPV for the entire project is still positive until scale M.

Figure 5.2 Relationship between NPV and Scale



The optimum scale of a project can also be determined by the use of the internal rate of return (IRR), assuming that each successive increment of investment has a unique IRR. If this condition is met, then the optimum scale for the facility will be the one at which the IRR for the incremental benefits and costs equal to the discount rate used to calculate the net present value of the project. This internal rate of return for the incremental investment required to change the scale of the project will be called marginal internal rate of return (MIRR) for a given scale of facility. The relationships between the IRR, the MIRR, and the NPV of a project are shown in Figure 5.3.

Figure 5.3 Relationships between MIRR, IRR, and the NPV



From Figure 5.3 we can observe that in a typical project, the MIRR from incremental investments will initially rise as the scale is increased, but will soon begin to fall with further expansions. This path of the MIRR will also cause the IRR to rise for the initial ranges of scale and then to fall. At some point the IRR and the MIRR must be equal and then change their relationship to each other. Prior to S_1 in Figure 5.3 the MIRR of the project is greater than the IRR: here expansions of scale will cause the overall IRR of the project to rise. At scales beyond S_1 , MIRR is less than IRR: in this range, expansions of scale will cause the overall IRR to fall.

The scale where the $IRR=MIRR$ is always the scale at which the IRR is maximized. However, it is important to note that this is not the scale at which the net present value of the project is likely to be maximized. The NPV of a project obviously depends on the discount rate. Only when the relevant discount rate is precisely equal to the maximum IRR will S_1 be the optimal scale. If the relevant discount rate is lower, it pays to expand the project's scale up to the point where MIRR is equal to the discount rate. As shown for the case when the discount rate is 10 percent, this scale yields the maximum net present value at a scale of S_2 in Figure 5.3.

To illustrate this procedure for the determination of the optimal scale of a project, let us consider the construction of an irrigation dam which could be built at different heights. Because of the availability of water we would expect that expansions of the scale of the dam would reduce the overall level of utilization of the facilities when measured as a proportion of its total potential capacity. The information is provided in Table 5-1.

Table 5.1
Determination of Optimum Scale of Irrigation Dam

Time	0	1	2	3	4	5	...		
Scales	Costs	Benefits						NPV	IRR
S ₀	-3,000	250	250	250	250	250	...	-500	0.083
S ₁	-4,000	390	390	390	390	390	...	-100	0.098
S ₂	-5,000	540	540	540	540	540	...	400	0.108
S ₃	-6,000	670	670	670	670	670	...	700	0.112
S ₄	-7,000	775	775	775	775	775	...	750	0.111
S ₅	-8,000	865	865	865	865	865	...	650	0.108
Time	0	1	2	3	4	5	...		
Changes in Scales	Incremental							Change in NPV	MIRR
	Costs	Benefits							
S ₀	-3,000	250	250	250	250	250	...	-500	0.083
S ₁ - S ₀	-1,000	140	140	140	140	140	...	400	0.140
S ₂ - S ₁	-1,000	150	150	150	150	150	...	500	0.150
S ₃ − S ₂	-1,000	130	130	130	130	130	...	300	0.130
S ₄ − S ₃	-1,000	105	105	105	105	105	...	50	0.105
S ₅ − S ₄	-1,000	90	90	90	90	90	...	-100	0.090

Notes: Discount rate (opportunity cost of funds) = 10%.

The depreciation rate of the dam is assumed at zero.

In this example we can calculate the Δ NPV for each scale of the dam from S₀ to S₅. Thus, the Δ NPV for S₀ is -500; for S₁-S₀ it is 400; for S₂-S₁ 500; for S₃-S₂ 300; for S₄-S₃ it is 50; and for S₅-S₄ it is -100.¹ If we use the above rule for determining the optimum scale, we would choose scale S₄ because beyond this point additions to scale add negatively to the overall NPV of the project. At scale S₄ we find that the net present value of the project is +750.0. At a scale of S₃ it is +700; and at S₅ it is +650. Therefore, net present value is maximized at S₄. If the project is expanded beyond scale 4 we find that the net present value begins to fall even though at a scale of S₅ the IRR of the entire project is still 0.108 (which is greater than the discount rate of 0.10). However, the MIRR is only 0.09, placing the marginal return from the last addition to scale below the discount rate.

¹ For perpetuity, the internal rate of return can be calculated as the ratio of annual income to initial investment.

5.3 Timing of Investments

One of the most important decisions to be made in the process of project preparation and implementation is the determination of the appropriate time for the project to start. This decision becomes particularly difficult for large indivisible projects such as infrastructure investments in roads, water systems and electric generation facilities. If these projects are built too soon, a large amount of idle capacity will exist. In such cases the foregone return (that would have been realized if these funds had been invested elsewhere) might be larger in value than the benefits gained in the first few years of the project's life. On the other hand, if a project is delayed too long, then shortages of goods or services will persist and the output foregone will be greater than the alternative yield of the funds involved.

Whenever the project is undertaken too early or too late, its NPV will be lower than what it could have been if it had been developed at the right time. The NPV of such projects may still be positive but it will not be at its maximum.

The determination of the correct timing of investment projects will be a function of how future benefits and costs are anticipated to move in relation to their present values. The situations where timing of investment projects becomes an important issue can be classified into four different cases. They are described as follows:

- a) The benefits of the project are a continuously rising function of calendar time, but investment costs are independent of calendar time.
- b) The benefits and investment costs of the project are rising with calendar time.
- c) The benefits are rising and then declining with calendar time while the investment costs are a function of calendar time.
- d) The benefits and investment do not change systematically with calendar time.

Case A: Potential Benefits are a Rising Function of Calendar Time

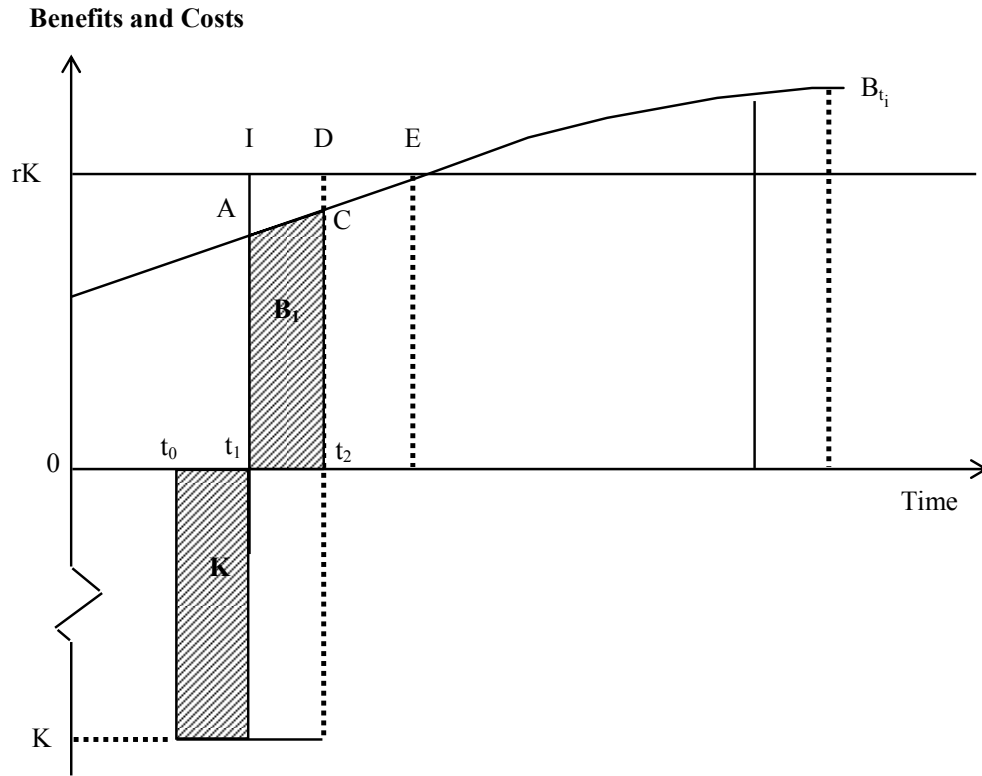
This is the case in which the benefits net of operating costs are continuously rising through

time and costs do not depend on calendar time. For example, the benefits of a road improvement rise because of the growth in demand for transportation between two or more places. It can thus be expected that as population and income grow, the demand for the road will also increase through time.

If the project's investment period ends in period t we assume its net benefit stream will start the year after construction, and will rise continuously thereafter. This potential benefit stream can be illustrated by the curve $B(t)$ in Figure 5.4. If construction were postponed from t_1 to t_2 , lost benefits amount to B_1 , but the same capital in alternative uses yields rK , where K denotes the initial capital expenditure and r denotes the opportunity cost of capital for one period. Postponing construction from t_1 to t_2 thus yields a net gain of AIDC. Similarly, postponing construction from t_2 to t_3 yields a net benefit of CDE.

In this situation the criterion to ensure that investments are undertaken at the correct time is quite straightforward. If the present value of the benefits that are lost by postponing the start of the project from time period t to $t+1$ is less than the opportunity cost of capital multiplied by the present value of capital costs as of period t , then the project should be postponed because the funds would earn more in the capital market than if they were used to start the project. On the other hand, if the foregone benefits are greater than the opportunity cost of the investment, then the project should proceed. In short, if $rK_t > B_{t+1}$, then postpone the project; if rK_t less than B_{t+1} , then undertake the project. Here t is the period in which the project is to begin, K_t is the present value of the investment costs of the project as of period t , and B_{t+1} is the present value of the benefits lost by postponing the project one period from t to t_1 .

**Figure 5.4 Timing of Projects:
Benefits are Rising but Investment Cost are Independent of Calendar Time**



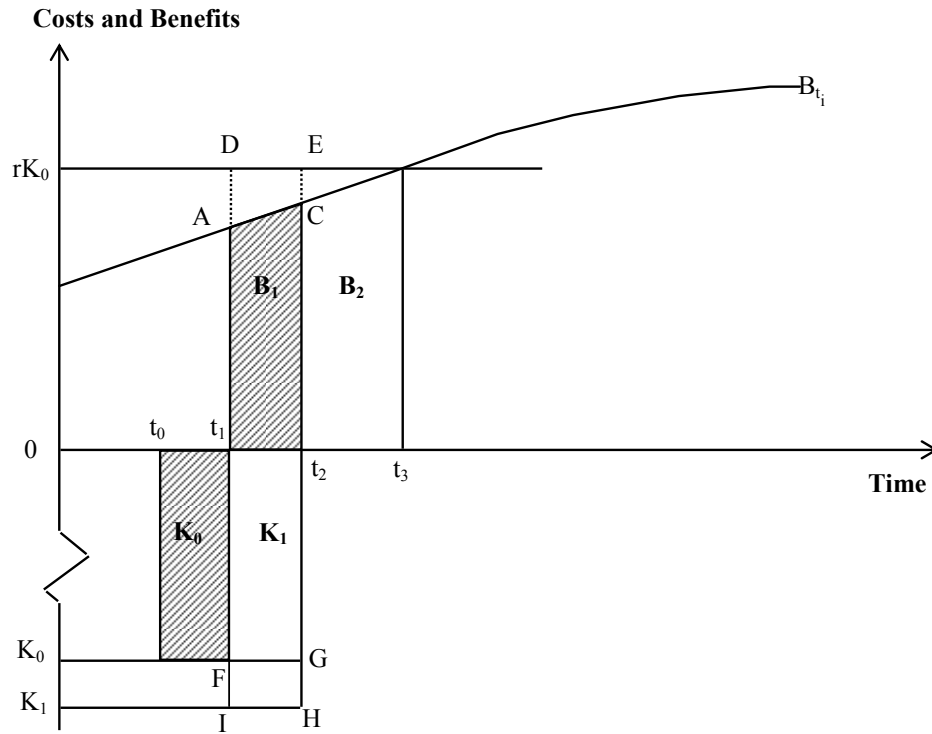
Rules:

$rK_t > B_{t+1}$	$\Rightarrow$	Postpone
$rK_t < B_{t+1}$	$\Rightarrow$	Start

Case B: Both Investment Costs and Benefits are a Function of Calendar Time

In this case, as illustrated in Figure 5.5, the investment costs and benefits of a project will grow continuously with calendar time. Suppose the capital cost is K_0 when the project is started in period t_0 and the costs will become K_1 if it is started in period t_1 . The change in investment costs must be included in the calculations of optimum timing. When the costs of constructing a project are bigger in period 1 than in period 0, there is an additional loss caused by postponement equal $(K_1 - K_0)$, as shown by the area FGHI in Figure 5.5.

**Figure 5.5 Timing of Projects:
Both Benefits and Investment Costs are a Function of Calendar Time**



Rules: $rK_t > B_{t+1} + (K_{t+1} - K_t) \Rightarrow$ Postpone
 $rK_t < B_{t+1} + (K_{t+1} - K_t) \Rightarrow$ Start

In case (a) when benefits are a positive function of calendar time, the decision rule for the timing of investments is to postpone if $rK_0 > B_1$, and to proceed as soon as $B_{t+1} > rK_t$. Now when the present value of the investment costs changes with the timing of the starting date, the rule is slightly modified. It becomes: if $rK_0 > [B_1 + (K_1 - K_0)]$, then postpone the project, otherwise undertake the project. The term, $(K_1 - K_0)$, represents the savings of the increase in capital costs by commencing the project in t_0 instead of t_1 . The rule shows a comparison of the area t_1DEt_2 with t_1ACt_2 plus FGHI. Hence, if investment costs are expected to rise in the future, it will be optimal for the project to be undertaken earlier than if investment costs remain constant over time.

Case C: Potential Benefits Rise and Decline According to Calendar Time

In the case where the potential benefits of the project are also a function of calendar time, but they are not expected to grow continuously through time, at some date in the future they are expected to decline. For example, the growth in demand for a given type of electricity generation plant in a country is expected to continue until it can be replaced by a cheaper technology. As the alternative technology becomes more easily available and cheaper, it is expected that the demand faced by the initial plant will decline through time. If the net benefits from an electricity generation plant are directly related to the volume of production it generates, we would expect that the pattern of benefits would appear similar to $B(t)$ in Figure 5.6.

If the project with present value of costs of K_0 is undertaken in period t_0 , its first year benefits will fall short of the opportunity cost of the funds shown by the area ABC. The correct point to start the project is t_1 when $rK_{t_1} < B_{t_2}$ and if the following project's NPV measured by the present value of the area under the $B(t)$ curve minus K_1 is positive:

$$NPV = -K/(1+r) + \sum_{t=2}^n [B_t/(1+r)^t]$$

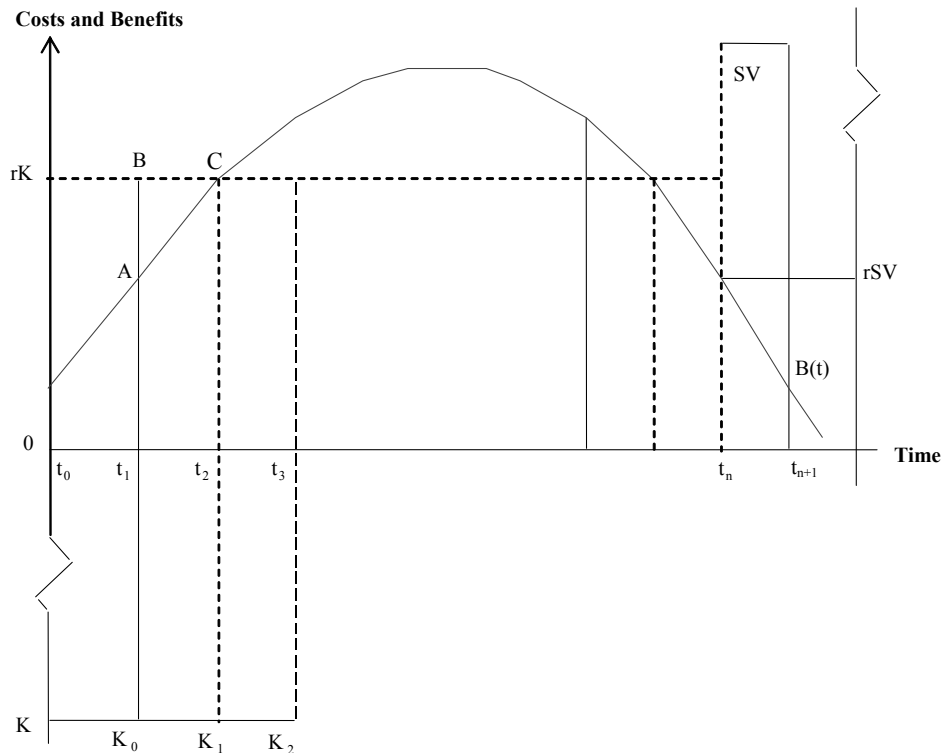
It is obviously essential that this net present value be positive in order for the project to be worthwhile.

The above formula assumes that the life of the project is infinite or that after some time its annual benefit flows fall to zero. In stead of lasting for its anticipated lifetime, the project could be abandoned at some point in time with the result that a one-time benefit is generated, equal to its scrap value, SV . In this case, it only pays to keep the project in operation so long as $B_{t_{n+1}} > rSV_{t_n} - \Delta SV_{t_{n+1}}$ so it would make sense to stay in business during t_{n+1} . If $B_{t_{n+1}} < rSV_{t_n} - \Delta SV_{t_{n+1}}$, it would make more sense to shut down operations at the end of t_n .

In practice, there are five special cases regarding scrap value and change in scrap value of a project:

- $SV > 0$ and $\Delta SV < 0$, e.g., machinery;
- $SV > 0$ but $\Delta SV > 0$, e.g., land;
- $SV < 0$ but $\Delta SV = 0$, e.g., a nuclear plant;
- $SV < 0$ but $\Delta SV > 0$, e.g., severance pay for workers; and
- $SV < 0$ and $\Delta SV < 0$, e.g., clean-up costs.

**Figure 5.6 Timing of Projects:
Potential Benefits Rise and Decline with Calendar Time**



Rules: Start if $rK_{t_i} < B_{t_{i+1}}$

Stop if $rSV_{t_n} - B_{t_{n+1}} - \Delta SV_{t_{n+1}} > 0$

$$\text{Do project if: } NPV^{t_i} = \sum_{i=t_{i+1}}^{t_n} \frac{B_{t_{i+1}}}{(1+r)^{t_{i+1}}} - K_{t_i} + \frac{SV_{t_n}}{(1+r)^{t_n-t_i}} > 0$$
$$\text{Do not do project if: } NPV^{t_i} = \sum_{i=t_{i+1}}^{t_n} \frac{B_{t_{i+1}}}{(1+r)^{t_{i+1}}} - K_{t_i} + \frac{SV_{t_n}}{(1+r)^{t_n-t_i}} < 0$$

In general, we should undertake a project if the following condition is met:

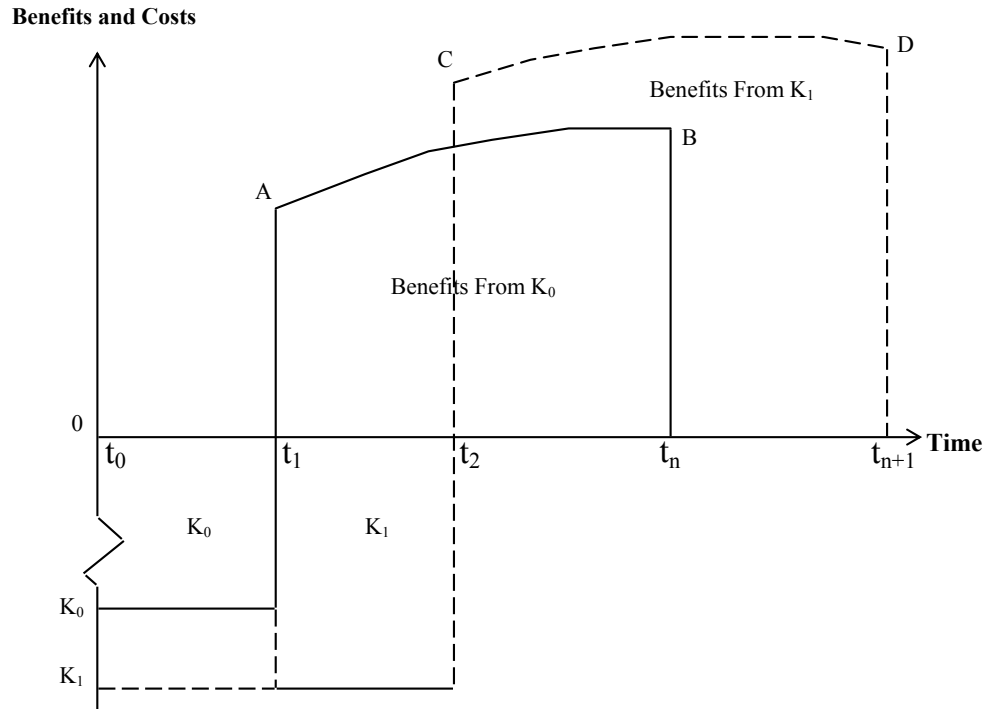
$$NPV^{t_i} = \sum_{i=t_{i+1}}^{t_n} \frac{B_{t_{i+1}}}{(1+r)^{t_{i+1}}} - K_{t_i} + \frac{SV_{t_n}}{(1+r)^{t_n-t_i}} > 0 \quad (5.1)$$

If this condition can not be met, and if $NPV^{t_i} = \sum_{i=t_{i+1}}^{t_n} \frac{B_{t_{i+1}}}{(1+r)^{t_{i+1}}} - K_{t_i} + \frac{SV_{t_n}}{(1+r)^{t_n-t_i}} < 0$, then we should not undertake the project.

Case D: Both Costs and benefits do not change systematically with calendar time

This is perhaps the most common situation where there is no systematic movement in either costs or benefits with respect to calendar time. As illustrated in Figure 5.7, if a project is undertaken in period t_0 its profile begins with investment costs of K_0 followed by a stream of benefits shown as the area t_1ABt_n . Alternatively, if it is postponed one period investment costs will be K_1 and benefits will be t_2CDt_{n+1} . In this case the optimal date to start the project is determined by estimating the net present value of the project in each instance and choosing the time to start the project which yields the greatest net present value.

**Figure 5.7 Timing of Projects:
Both Benefits and Costs do not Change Systematically with Calendar Time**



It is important to note that in determining the timing of the project, the date to which the net present values are calculated must be the same for all cases even though the period in which the projects are to be initiated varies.

5.4 Adjusting for Different Lengths of Life

When there is no budget constraint and when a choice must be made between two or more mutually exclusive projects, then investors seeking to maximize net worth should select the project with the highest NPV. If the mutually exclusive projects are expected to have continuous high returns over time then it is necessary to consider the length of life of the two or more projects. The reason for wanting to ensure that mutually exclusive projects are compared over the same span of time is to give them the same opportunity to accumulate value over time. One way to think about the NPV is as an economic rent that is earned by a

fixed factor of production. In the case of two mutually exclusive projects, for example, the fixed factor could be the building site, a right-of-way, or a license. That fixed factor should have the same amount of time to generate economic rents regardless of which project is chosen. What is required is a reasonable method of equalizing lengths of life that can be applied. This is elaborated with the help of the following two illustrations.

Illustration 1

Consider two mutually exclusive projects with the same scale of investment, a three-year project A and a four-year project B, that have the net cash flows as shown in Table 5.2. All the net cash flows are expressed in thousands of dollars and the cost of capital is 10%.

Table 5.2 Net Cash Flows for Projects with Different Length of Life

Time Period	t_0	t_1	t_2	t_3	NPV@10%
Net Cash Flows of Project A	-10,000	6,000	6,000		410
Net Cash Flows of Project B	-10,000	4,000	4,000	4,750	500

If we were to overlook the differences in the lengths of life, then we would select project B because it has the higher NPV. To do so, however, would run the risk of rejecting the potentially better project A with the shorter life.

One approach to this problem is to determine whether we might be able to repeat the projects a number of times (not necessarily the same number of times for each project) in order to equalize their lives. To qualify for this approach, both projects must be supra-marginal (i.e., have positive NPVs) and should be repeatable at least a finite number of times.

Assume that the two projects, A and B, above meet these requirements. If we were to repeat project A three times and project B twice, then both projects would have a total operating life of 6 years, as shown in Table 5.3.

Table 5.3 Net Present Value for Repeating Projects

Time	t_0	t_1	t_2	t_3	t_4	t_5	t_6
Project A's NPV for each repeat	410	–	410	–	410	–	410
Project B's NPV for each repeat	500	–	–	500	–	–	500

In year t_6 both projects can start up again, but there is no need to repeat this procedure. The construction of the repeated projects is initiated so as to maintain a level of service. For example, construction for the second project B begins in year t_3 so that it is ready to begin operations when the first project B stops providing service. Given the equal lengths of life for the repeated projects, they can now be compared on the basis of the net present value:

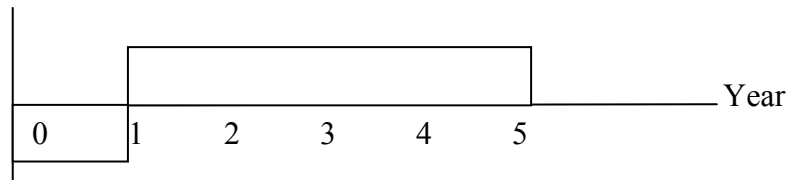
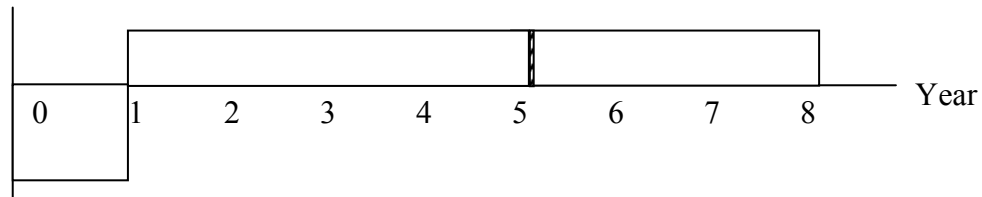
$$\text{NPV of Project A's repeats} = 410 + \frac{410}{(1.1)^2} + \frac{410}{(1.1)^4} = 1,029$$

$$\text{NPV of Project B's repeats} = 500 + \frac{500}{(1.1)^3} = 876$$

Given an equal opportunity to earn economic rents, project A has a higher overall NPV and should be considered the more attractive project.

Illustration 2

This example refers to the case when a choice is to be made between mutually exclusive projects representing different types of technology with different lengths of life.

Project I (Technology 1) $B_t - C_t$ **Project II (Technology 2)** $B_t - C_t$ 

How can we know which technology to choose using the NPV criterion? Suppose that the present value of the costs of project I [$PV_0(C_0^I)$] is \$100 and the present value of its benefits [$PV_0(B_{1-5}^I)$] is \$122. Similarly, the present value of the costs of Project II [$PV_0(C_0^{II})$] is \$200, and of its benefits [$PV_0(B_{1-8}^{II})$] is \$225. If we compare the NPVs of the two projects, it would appear that project II is preferred to project I, because the NPV of project II is \$25, whereas that of project I is only \$22.

However, since these two projects represent two different types of technology with different lengths of life, the NPV of project II is biased upward.

In order to make a correct judgment, we need to make them comparable by either adjusting the lengths of life or calculating annualization of net benefits. The first way could be to adjust project II to make it comparable to project I. The benefits for only the first five years of project II should be included, and its costs should be reduced by the ratio of the present value of benefits from year 1-5 to year 1-8. This is expressed as follows:

$$NPV_0^I = PV_0(B_{1-5}^I)$$

$$NPV_0^{II\text{Adj}} = PV_0(B_{1-5}^{II}) - PV_0(C_0^{II}) \left(\frac{PV_0(B_{1-5}^{II})}{PV_0(B_{1-8}^{II})} \right)$$

Plugging in the values of costs and benefits of the two projects in our example, we have:

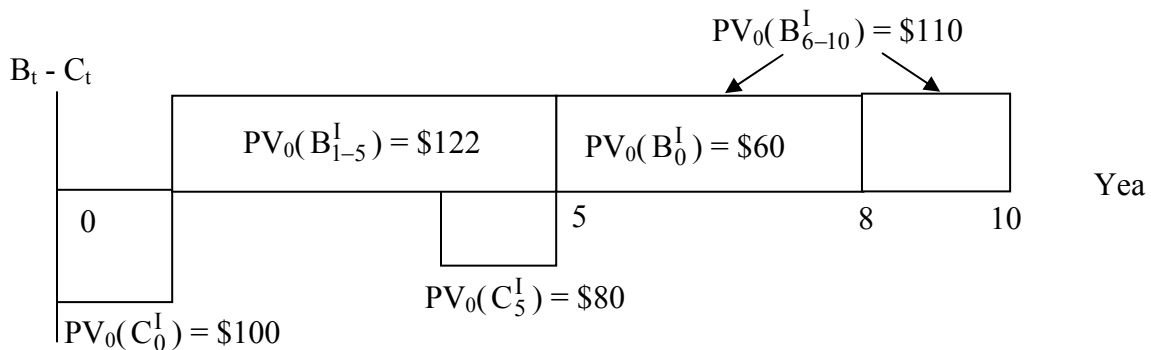
$$PV_0(B_{1-8}^{II}) = \$225, \quad PV_0(C_0^{II}) = \$200, \quad PV_0(B_{1-5}^{II}) = \$180.$$

Hence, the $NPV_0^I = \$122 - \$100 = \$22$, and

$$\text{the } NPV_0^{II\text{Adj}} = \$180 - \$200(180/225) = \$180 - \$160 = \$20.$$

After the adjustment, the NPV of project I is greater than that of project II which means that project I is better.

The second way to make the two projects comparable would be to adjust the length of project I. We need to calculate the NPVs of project I (adjusted) and project II. We will adjust the NPV of project I by doubling its length of life. Then the benefits of years 6-8 are added to the benefits of years 1-5. The costs are increased by the value of the costs to lengthen the project to year 8, which is the present value of the costs in year 5, reduced by the ratio of the benefits of years 6-8 to the benefits of years 6-10.



This adjustment can be expressed as follows:

$$NPV_0^{IAdj} = PV_0(B_{1-5}^I) - PV_0(C_0^I) + PV_0(B_{6-8}^I) - PV_0(C_5^I) \left(\frac{PV_0(B_{6-8}^I)}{PV_0(B_{6-10}^I)} \right)$$

Plugging in the present values of costs and benefits of the two projects in our example, we have:

$$\text{the } NPV_0^{IAdj} = \$122 - \$100 + \$60 - \$80(60/110) = \$38.36, \text{ and}$$

$$\text{the } NPV_0^{II} = \$225 - \$200 = \$25.$$

Using this method, we still have the NPV of project I greater than that of project II. Therefore, project I is preferred to project II.

The third way is to compare annualization of the net benefits of two projects. For project I, the present value of net benefits is \$22 over a 5-year period. The annualized value of the benefits can be calculated as follows:²

$$\text{Annualized Value}^I = [\$22 \cdot 0.10] / [1 - (1 + 0.10)^{-5}] = \$5.80$$

For project II, the present value of net benefits is \$25 over a 8-year period. The annualized value of the benefits is:

$$\text{Annualized Value}^{II} = [\$25 \cdot 0.10] / [1 - (1 + 0.10)^{-8}] = \$4.69$$

Again, the higher present value of the net benefits for project II than project I is due to a longer time horizon. When they are normalized in time period, it is shown that project I is in fact preferred.

5.5 Projects with Interdependent and Separable Components

Often an investment program will contain several interrelated investments within a single project. It has sometimes been suggested that in such integrated projects it is correct to evaluate the project as a whole and to bypass the examination of each of the sub-

² European Commission, *Impact Assessment Guidelines*, (June 15, 2005), and *Annexes to Impact*

components. This argument is generally not correct. The analyst should attempt to break the project down into its various components and examine the incremental costs and benefits associated with each of the components to determine whether it increases or decreases the NPV of the project.

Suppose the task is to appraise a project to build a large storage dam, planned to provide hydroelectric power, irrigation water, and recreational benefits. Upon first examination of this project it might appear that these three functions of the dam are complementary, so that it would be best to evaluate the entire project as a package. However, this is not necessarily the case. The irrigation water might be needed at a different time of the year than the peak demand for electricity. The reservoir might be empty during the tourist season if the water were used to maximize its value in generating electricity and providing irrigation. Therefore, to maximize the NPV of the whole package, it may mean that efficiency of some of the components will be reduced. In this case the overall project might be improved if one or even two of the components were dropped from the investment package.

To appraise such an integrated investment package we should begin by evaluating each of the components as an independent project. Thus, the hydroelectric power project would be evaluated separately. The technology used in this case would be the most appropriate for this size of an electricity dam without considering its potential as a facility for either irrigation or recreational use. Similarly, the use of this water supply in an independent irrigation project or an independent recreational development should be appraised on its own merits.

Next, the projects should be evaluated as combined facilities such as an electricity-cum-irrigation project or an electricity-cum-recreational project or an irrigation-cum-recreational project. In each of these combinations the technology and operating program should be designed to maximize the net benefits from the combined facilities. Lastly, the combined electricity, irrigation and recreational project are evaluated. Again, the technology and operating plans will have to be designed to maximize the net benefits from the combined

facilities. These alternatives must now be compared to find the one that yields the maximum NPV. Frequently, the project which ends up with the greatest NPV is one containing fewer components than was initially proposed by its sponsor.

A common investment problem of the type which involves separable component projects arises when a decision is being made as to whether or not existing equipment should be replaced. When faced with this decision, there are three possible courses of action:

- (a) Keep the old asset and do not buy the new asset now;
- (b) Sell the old asset and purchase the new one; or
- (c) Keep the old asset and in addition buy the new one.

Let us denote the present value of all future benefits that could be generated by the old asset (evaluated net of operating cost) by B_o and the liquidation or scrap value of the old asset if sold as SV_o . We will express the present value of future benefits (net of operating costs) from the new asset as B_n and the present value of the investment costs for the new asset as C_n . Also, the combined benefits from the use of the old and new asset together will be denoted as B_{n+o} .

The first thing that has to be done is to appraise all of the three alternatives to determine which of them are feasible, i.e., which of the three generate positive net present values. These comparisons are as follows:

- (1) In order that alternative (a) be feasible, it is necessary that the present value of the future benefits from the old asset exceeds its liquidation value, i.e., $B_o - SV_o > 0$.
- (2) In order for alternative (b) to be feasible, it is necessary that the present value of the future benefits from the new asset be bigger than the present value of its investment costs, i.e., $B_n - C_n > 0$.
- (3) For alternative (c) to be feasible, the total benefits produced by both assets combined must be greater than the costs of the new investment plus the liquidation value of the

old asset. In this case the old asset is retained to be used along with the new asset.

This is expressed as $B_{n+o} - (C_n + SV_o) > 0$.

If each of the alternatives is feasible, then we must compare them to determine which component or combination yields the greatest NPV. To determine whether or not to replace the old asset with the new one, we inquire whether $(B_n - B_o) - (C_n - SV_o) > 0$. If this expression is less than zero, then we surely will not exchange assets, but would still be willing to retain the old asset while purchasing the new one if $(B_{n+o} - B_n) - SV_o > 0$. This condition for retaining the two assets amounts to each of them justifying itself as the marginal asset. Alternatively, if $(B_n - B_o) - (C_n - SV_o) < 0$ and $(B_{n+o} - B_o) - C_n < 0$, then we should simply continue using the old facilities without any new investment. Finally, if the conditions are $(B_n - B_o) - (C_n - SV_o) > 0$ and $(B_{n+o} - B_n) - SV_o < 0$, then we should replace the old asset with the new one.

One way to describe the comparisons that have just been made is to define $(B_{n+o} - B_o)$ as $B_{n/o}$ and $(B_{n+o} - B_n)$ as $B_{o/n}$. In this notation $B_{n/o}$ is the incremental benefit of the new asset in the presence of the old, and $B_{o/n}$ is the incremental benefit of the old asset in the presence of the new. The condition that is required for both assets to be present in the final package is that both $B_{n/o} > C_n$ and $B_{o/n} > SV_o$. This means each component must justify itself as the marginal item in the picture.

This same principle governs in all cases where one has to deal with separable components of a project. Each separable component must justify itself as a marginal or incremental part of the overall project.

The careful examination of the alternative components of a potentially integrated project is thus an important task in the preparation and appraisal phase of a project. Failure to do so may mean that potentially valuable projects are not implemented because they were evaluated as part of a larger unattractive package. On the other hand, wasteful projects might get implemented because they have been included in a larger integrated project which as a whole is worthwhile, but could be improved if the wasteful components were eliminated.

Complementarity and Substitutability among Projects

Once the door has been opened to interrelations among projects, a substantial gamut of possibilities emerges. It is instructive to examine these possibilities in detail. Denoting PVB as present value of benefits and PVC as present value of costs, we have the following cases:

$PVB_I + PVB_{II} = PVB_{I+II}$ Projects I and II are independents on the benefit side;

$PVB_I + PVB_{II} > PVB_{I+II}$ Projects I and II are substitutes on the benefit side;

$PVB_I + PVB_{II} < PVB_{I+II}$ Projects I and II are complements on the benefit side;

$PVC_I + PVC_{II} = PVC_{I+II}$ Projects I and II are independents on the cost side;

$PVC_I + PVC_{II} > PVC_{I+II}$ Projects I and II are complements on the cost side;

$PVC_I + PVC_{II} < PVC_{I+II}$ Projects I and II are substitutes on the cost side.

We will not deal with independent projects here. Examples would be a spaghetti factory in San Francisco and a highway improvement on Long Island. One has essentially nothing to do with the other. We have already dealt with a case where projects are substitutes on the benefit side. It is impossible for a multipurpose dam to generate, as a multipurpose project, the sum total one could get of the benefits of the same project (e.g., a dam), independently maximized for each separate purpose, were added together. Thus, multipurpose dams invariably entail substitution among the separate purposes.

Complementarity on the benefit side is almost easy to deal with. An automobile will not function on three wheels or without a carburetor. Hence the marginal benefit of adding the fourth wheel, or the carburetor, is enormous. A more subtle case of complementarity on the benefit side, well known in the literature of economics, is that of an apiary project together with an orchard. The presence of the orchard enhances the benefits of the apiary; the presence of the bees also enhances the value of the orchard.

Whereas the separate purposes of multipurpose dams are invariable substitutes on the benefit

side, they are practically always complements on the cost side. To build one dam to serve several purposes will almost always cost less than the sum total of the two or more costs of building (at least hypothetically) separate dams to serve each of the separate purposes.

Cases of substitutability on the cost side are a bit harder to come by, but they clearly exist. A dam project that will produce a larger lake will clearly be competitive with a highway whose natural route would cross the area to be flooded. The total costs of the two projects together will exceed the sum of the costs of the two, considered above. Similarly, a project to urbanize an area will likely compound the costs of a highway project going through that area.

Altogether, one must be alert to the possibilities of substitution and complementarity between and among projects. The underlying principle is always the same: maximize net present value. Its corollary is precisely the principle of separable components, previously stated. Each separable component must justify itself as the marginal one. This becomes a problem where issues of substitutability are involved, rarely so in cases of complementarity on both sides (benefits and costs). Perhaps the most interesting cases are those (like multipurpose dams) where complementarity on one side (in this case the cost side) has to fight with substitutability on the other.

5.6 Conclusion

Timing and scale of projects are often important consideration of project evaluation. This chapter has discussed the issues and presented some decision rules for projects according to the net present value criterion. Project analysts are also often to face an issue of choosing highly profitable mutually exclusive projects with different lengths of life. We have provided alternative approaches to either adjust the costs or benefits or annualize the benefits in making a choice between mutually exclusive projects.

In reality, an investment often contains several interrelated investment, either substitute or complementary. We have demonstrated that the concept of the net present value of the

CHAPTER 5:

project's benefits and costs can provide a powerful tool for selecting project with single component or combination of components.

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